PRODUCTION PLANNING & CONTROL, 1998, VOL. 9, NO. 5, 504–510

# JIT sequencing for mixed-model assembly lines with setups using Tabu Search

P. R. McMULLEN

**Keywords** Tabu Search, JIT, heuristics, assembly lines

**Abstract.** This research presents a heuristic for sequencing mixed-model production schedules for assembly lines when JIT production is an objective, and setup requirements are present. The heuristic examines a sequence and determines an objective function value based upon the parts usage rate (Miltenburg 1989) and the number of setups involved. This sequence is then altered in the hope of finding a better sequence in terms of the objective function via Tabu Search. This technique is applied to several problems, and the resulting sequences are simulated to determine production performance measures of production makespan, system time and average WIP inventory level. The experiment shows that the multiple objectives of minimizing both parts usage rate and required setups can be addressed provided management has an understanding of the relative importance of usage rate and setups for their specific application.

## 1. Introduction

As JIT systems have gained more popularity in industry, system flexibility has generally improved. Average WIP inventory levels have decreased, as has the amount of time in-process units spend in the system (system flowtime). The challenge of successfully implementing a JIT system is formidable for many reasons—the particular reason that this paper concerns itself with is the product sequencing issue for mixed-model assembly lines when setup times are not assumed to be negligible.

Product sequencing is important because the sequence determines the rate at which the raw materials are used for production. When several different products are to be made on an assembly line, this usage rate of materials is especially sensitive to the production sequence. Because the material usage rate is sensitive to the production sequence, considerable effort has gone into development of techniques intended to minimize this material usage rate. Monden (1983), Miltenburg (1989), Wantuck (1989), and Sumichrast and Russell (1990) have all addressed the sequencing issue with regard to minimization of the material usage rate.

While keeping the usage of materials as constant or as ‘smooth’ as possible is of extreme importance, it is not the only objective to be considered. When the required changeover time between different products is not negligible, an effort should be made to minimize the amount of total changeover time as well. The work previously cited utilizes the underlying assumption of negligible changeover times. This research utilizes the assumption that changeovers between differing products are not negligible. As a result, this research is dedicated to determining JIT production sequences for mixed-model assembly lines which provide reasonable levels of both material usage rates and setups (or product changeovers).

At this point, it is necessary to elaborate on these setups times which are considered non-negligible. While this specific research focuses on non-negligible setups times between different products, the actual setups times are still assumed to be a relatively small portion of the actual time required to process the product requiring the setup

---

*Authors:* Patrick R. McMullen, University of Maine, Maine Business School, Orono, Maine 04469-5723, USA.

Patrick R. McMullen is an Assistant Professor at the Maine Business School at the University of Maine. He teaches courses in Production and Operations Management, Operations Research and Quantitative Analysis to both undergraduate and MBA students. He also teaches courses in Business Forecasting/Econometrics and Game Theory at the Harvard University Summer School. He received his Ph.D. in Production and Operations Management from the University of Oregon in 1995. His research interests are in the applications of Operations Research techniques in the design of production systems. He has published articles dealing with assembly lines and Data Envelopment Analysis.

0953-7287/98 $12·00

for the resource of interest (20% is the proportion used for this research experiment). When setup times required between differing products are significantly longer, an assembly line is probably not the best production layout to implement—perhaps more of a job shop layout would be best.

To attain production sequences which address this multiple objective type of problem, a technique known as Tabu Search is exploited. Tabu Search is a powerful heuristic used to provide near optimal solutions for combinatorial optimization problems. A production sequencing problem with the previously stated objectives can be thought of as a combinatorial optimization problem because there are many different possible sequences to choose from. Here, Tabu Search is used in finding JIT sequences for mixed-model assembly lines which address the objectives of minimization of the material usage rate and required setups for several different mixed-model assembly line scheduling problems. Glover (1990, 1993) offers informative explanations of Tabu Search.

For each of these scheduling problems, five different solutions are found—each solution places a different emphasis on the two objectives. These JIT sequences found using Tabu Search for the multiple objective problems are then used as schedules for simulated production runs. The simulated production runs provide output performance measures of average work-in-process (WIP) inventory level, average amount of time a unit spends in the system (flow-time) and the amount of time required to complete the entire production schedule (makespan).

The simulated production runs are used as a database for an experimental design intended to shed light on which of the five different solutions for each problem perform best with regard to the output measures of interest.

The following sections detail the Tabu Search heuristic, the sequencing problem at hand and the experimental design. General comments regarding using Tabu Search for these types of problems are also offered.

## 2. Tabu Search and sequencing

Consider the following list of products needing to be manufactured via an assembly line (table 1):

Table 1. Example sequencing problem.

| Product | Units Required |
|---|---|
| A | 4 |
| B | 2 |
| C | 1 |

A total of seven products needs to be manufactured $(4 + 2 + 1 = 7)$, for a total of seven positions in the sequence. There are 7! ways to order the seven positions in the sequence, with 4! of these ways redundant because all of the product As are the same, 2! of these ways redundant because all of the product Bs are the same, and 1! of these ways redundant for the lone unit of product C. This scenario results in 105 different sequencing possibilities:[1]

$$\frac{7!}{4!2!1!} = 105 \qquad (1)$$

For such a small sequencing problem, 105 possibilities seems quite large. A problem twice this size (8 units of A, 4 of B and 2 of C) results in 45 045 possible sequences. It then becomes quite clear that this problem is combinatorial, and that finding a 'best' sequence via some type of mathematical programming technique is basically impossible—especially for larger problems.

Because of the combinatorial nature of such sequencing problems, a search technique needs to be employed to attain a 'near-optimal' condition with respect to an objective function. Tabu Search is the technique chosen for this multiobjective sequencing problem. What sets Tabu Search apart from other search techniques is that it utilizes a short term memory component of previous solutions which prevents 'cycling', which can in turn result in being trapped at local optima, thereby preventing finding an optimal (or near-optimal) solution. Tabu Search takes an initial solution and makes changes to this solution during an iterative process. As changes are made, they are recorded on a 'tabu list', which is simply a listing of the most recent changes, or 'moves'. If a move under consideration appears on the tabu list, the move is forbidden (tabu) unless its objective function value satisfies what is known as aspiration criteria—which is explained in more detail later. This basic procedure is repeated until user-specified stopping criteria are met.

Prior to a detailed discussion of Tabu Search, relevant variables and objective function issues are introduced. Consider the following variables: $U$ = usage rate of a production sequence; $S$ = number of setups in a production sequence; $a$ = number of products to be assembled in the line; $D_T$ = total number of units for all products or total demand—positions in sequence; $d_i$ = demand for product $i$, $i = 1, 2, \ldots, a$; $x_{i,k}$ = total number of units of product $i$ produced over stages $1 - k$, where $k = 1, 2, \ldots, D_T$.

The number of setups for a specific sequence is computed as follows:

---

[1] The combinations counting rule was used to determine the number of sequencing possibilities for equation (1): $= n!/k!(n - k)!$

$$S = \sum_{k=1}^{D_T} s_k, \tag{2}$$

The usage rate for a specific sequence is computed as follows:

$$U = \sum_{k=1}^{D_T} \sum_{i=1}^{a} \left( x_{i,k} - k \cdot \frac{d_i}{D_T} \right)^2 \tag{3}$$

where $s_k = 1$ if the product in position $k$ is different from the product in position $k - 1$, or 0 otherwise. An objective function value for the production sequence is then determined, which is a composite measure of equations (2) and (3), where $w_U$ is the weight placed upon the usage rate and $w_S$ is the weight placed upon the number of setups. The composite objective function is then:

$$\text{Min: } Z = w_S S + w_U U \tag{4}$$

which is determined for all sequences.

The problem in table 1 would have the following sequence if minimization of setups were the only objective: A-A-A-A-B-B-C, with three setups and a usage rate of 11.714. The problem in table 1 would have the following sequence if minimization of usage rate were the only objective: A-B-A-C-A-B-A, with seven setups and a usage rate of 1.714.

### 2.1. *Initialization*

To commence the Tabu Search procedure, system parameters are initialized. The tabu list length is specified by the user. This is the number of most recent solutions that will be checked against the current test solution to determine if the current test solution is making a move that is considered forbidden, or tabu. The user also specifies the sample size of solutions evaluated at each iteration ($n$). Specify the desired number of iterations ($N$) and initialize the iteration counter ($Iter$) to 1. Specify the desired limit of iterations made without the best solution being replaced ($B$), and initialize the number of iterations since the best solution has been replaced to 0 ($Biter$).

### 2.2. *Initial solution*

An initial solution is generated using Ding and Cheng's heuristic (1993), which minimizes the material usage rate of the sequence. This initial solution becomes the current solution as well as the best solution.

### 2.3. *Find new solution*

From the current solution, randomly select pairs of unique positions in the sequence until two unique positions with corresponding unique products are found. Then, perform a swap of these products into the randomly selected positions. For example, if position 4 accommodates product A and position 15 accommodates product D, then the swap will result in position 4 accommodating product D and position 15 accommodating product A. At this point, the objective function is updated via equations (2)–(4) to account for the recently performed swap. This process is repeated $n$ times to the current solution, where $n$ is the sample size established by the user. Of the $n$ solutions generated, the one with the lowest objective function value in equation (4) is selected.

### 2.4. *Check Tabu Status*

If the move resulting in the test solution selected in the previous step appears on the tabu list, then the move is considered tabu, or forbidden (referred to as a tabu solution), and its objective function value must be compared against the aspiration value of the solution appearing on the tabu list which made the same move as the tabu solution (Step 2.5). If the test solution is not considered tabu, the test solution becomes current and proceed to Step 2.6.

### 2.5. *Check aspiration value of corresponding solution on tabu list*

If the objective function value of the tabu solution is less than the aspiration value of the corresponding solution on the tabu list, the tabu status of the tabu solution is overridden, the solution becomes the current solution, it is added to the tabu list, and the aspiration value of the corresponding solution on the tabu list is replaced with the objective function value of the new current solution. Otherwise, the tabu status cannot be overridden and the user must return to finding a new solution (Step 2.3). The aspiration value (cost) of a solution is the lowest cost of all solutions arrived at from solutions with this cost. An informative description of the aspiration issue is provided by Bland and Dawson (1991).

### 2.6. *Update system parameters*

Examine the objective function value of the current solution against the objective function value of the best solution. If the objective function of the current solution

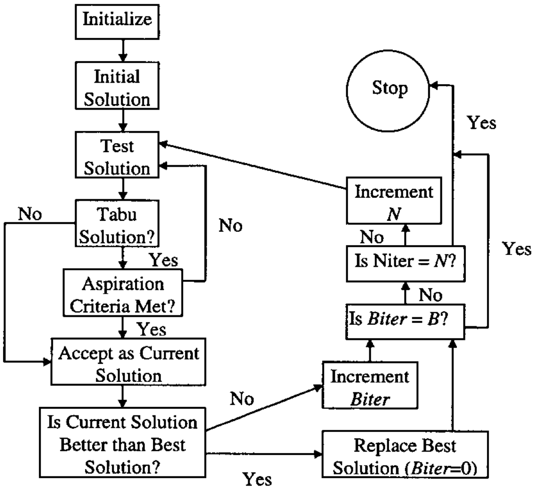

Figure 1. Flow-chart of Tabu search heuristic.

is less than that of the best solution, the best solution is replaced with the current solution—both sequence and objective function value, and the value of $\mathit{Biter}$ reset to 0. Otherwise, increment $\mathit{Biter}$ by 1. Regardless of the action taken, update the tabu list to account for the new current solution and proceed to the next step.

### 2.7. *Control*

If either the upper limit of iterations since the best solution has been updated ($B$) or the desired number of iterations ($N$) has been reached, the Tabu Search procedure is complete. Otherwise, the iteration counter ($\mathit{Iter}$) is incremented by 1 and find a new test solution (Step 2.3).

## 3. Sequencing heuristics used

To demonstrate how this Tabu Search procedure is used to address the JIT sequencing problem with setups, three different problem sets are examined (Sumichrast and Russell 1990—Appendix). Each problem set shows 10 different scenarios where sequencing is required. Within each problem set, there are varying degrees of product mix. In some situations, one or two products dominate the product mix, and in others, all (or most) of the products generally make equal contributions to the product mix.

Each of the 30 individual sequencing problems (three problem sets, 10 problems per set) are solved five times—each time employing a different objective function and heuristic. The different heuristics are as follows:

### 3.1. *Heuristic 1*

This heuristic sequences the products in such a way that the required number of setups is minimized. This sequencing can be performed by inspection—no heuristics are necessary. The objective function for this is as follows:

$$\text{Min: } z = s \tag{5}$$

### 3.2. *Heuristic 2*

The objective function for this heuristic is:

$$\text{Min: } z = u \tag{6}$$

This objective is addressed by using Ding and Cheng's heuristic (1993), which is a simplification of Miltenburg's work (1989). This heuristic minimizes the material usage rate.

### 3.3. *Heuristic 3*

Tabu Search is employed for this objective which sequences the products in such a way that a composite function of both number of setups and materials usage rate is minimized. The objective function is as follows:

$$\text{Min: } z = 14.2755s + 1u \tag{7}$$

The coefficients used for this objective function come from sampling. Several solutions were sampled across many problems and the coefficients were determined such that both the number of setups and the material usage rate made equal contributions to the objective function.

### 3.4. *Heuristic 4*

The heuristic used to address this objective also employs Tabu Search as in equation (7), but here the number of setups is weighted three times as much. In other words, both setups and usage rate are still considered, but now, minimizing the number of setups is three times as important as minimizing the usage rate.

This objective function is as follows:

$$\text{Min: } Z = 3 * 14.2755s + 1s \tag{8}$$

### 3.5. *Heuristic 5*

The heuristic used to address this objective also employs Tabu Search as in equation (7), but here the

Table 2. Tabu Search parameters.

| | Problem Set 1 | Problem Sets 2 and 3 |
|---|---|---|
| Sample size/iteration ($n$) | 10 | 20 |
| Tabu List length | 25 | 30 |
| Maximum iterations ($N$) | 200 | 250 |
| Iterations without improvement ($B$) | 20 | 25 |

usage rate is weighted three times as much. In other words, both setups and usage rate are still considered, but now, minimizing the usage rate is three times as important as minimizing the number of setups.

This objective function is as follows:

$$\text{Min: } Z = 14.2755S + 3U \tag{9}$$

Table 2 shows the Tabu Search parameters used for Heuristics 3–5.

## 4. Experimental design

As stated, each of the 30 different problems are solved by the five different heuristics. This yields a total of 150 production sequences. Each of these sequences is then used as a production schedule in conjunction with simulation so as to see which of the five heuristics perform best with regard to certain production-related measures.

### 4.1. *Simulation runs*

The input to the production simulation model is the production schedule, or sequence. The simulation model will use this schedule as an input to provide output values for some production-related measures. The output measures of most interest are the amount of time required to complete the entire production run (makespan), the average work-in-process inventory (WIP) level and the average amount of time a unit spends in the system (flowtime). Some of the important characteristics of this simulation model are as follows:

- A pull system is used.
- Parts scheduled for production are dispatched in a deterministic fashion.
- There are no notable differences in the raw materials required for the different parts (Miltenburg 1989).
- For each simulated production run, the actual production sequence is repeated 10 times for each of the 30 problems in the Appendix, so reasonable estimates of the outputs can be attained.
- Each unit is processed through seven different resources.
- The process time for each unit through each resource is assumed to be a normally distributed random variable with a standard deviation being 15% of its expected value.
- The expected process time for each different product through each resource is unique.
- When a setup is required, its expected duration is a normally distributed random variable with an expected duration of 20% of the expected process time for that particular resource requiring the setup, and its standard deviation is 15% of its mean.

The simulation run for each of the 150 production sequences is repeated 25 times, so reasonable estimates of the output measures can be attained, yielding a database of 3750 points. SLAMSYSTEM v4.6 and FORTRAN v5.1 user-written inserts were used to model the described production system.

### 4.2. *Research questions*

To provide some information as to which of the heuristics will be of most use to the decision-maker concerned with this particular sequencing issue, the following research questions were constructed:

- Do the five sequencing heuristics result in differing levels of the output measures?
- Which sequencing heuristics result in the best performance?

These research questions were addressed for each individual problem set in the Appendix.

## 5. Results

Prior to presenting general results, it should be noted that the two outputs of average WIP level and average flow time were both positively correlated with the output of makespan (0.953 and 0.931, respectively). As a result, they are not interpreted and are explained by makespan from this point forward—makespan then being the only simulation output interpreted. Along with interpreting the output measure of makespan, the inputs of setups and usage rate are also interpreted, so that an additional perspective can be gained for the different sequencing objectives.

Table 3. Test statistics for attributes of sequencing heuristics.

| | Problem Set 1 | Problem Set 2 | Problem Set 3 |
|---|---|---|---|
| Overall (Wilks' lambda) | 248.69 | 329.04 | 445.17 |
| Makespan | 505.36 | 275.58 | 996.27 |
| Setup requirement | 377.74 | 170.69 | 440.05 |
| Usage rate | 437.28 | 780.29 | 1289.79 |

When interpreting the inputs, the number of required setups for each sequence has been divided by total demand for each of the 150 problems. The total demand is the same as the number of positions in the associated sequence. Dividing the number of required setups by the number of positions in the sequence yields the percentage of time a setup is required. This is done to standardize the number of setups required across the three differing problem sizes used for this research. With that stated, it should also be noted that there is some correlation between the makespan and the percent of time that setups are required (0.775). Although some correlation exists, both measures will remain in the analysis, so that as rich an explanation as possible can be given.

Table 3 shows the F-test results of a MANOVA with ANOVA follow-ups to investigate what effect(s) the five sequencing heuristics had on makespan, setup requirement and usage rate.

In all cases, the sequencing heuristic had a significant effect on the performance measures (all $P$ values < 0.0001)—both an overall multivariate effect on all measures and univariate effects on the individual measures. To provide further details of this, table 4a–c shows the mean values (and standard deviations) for our three measures of interest organized by problem set.

## 6. Discussion of results

Each of the five sequencing heuristics provides some unique performance attributes. Heuristic 1 results in a minimum number of setups which of course minimizes the makespan because of fewer setups. Unfortunately, this also results in a very poor material usage rate—products are essentially being made in batches, and no flexibility is realized. Heuristic 2 provides just the opposite results—very poor makespan performance due to the frequent number of required setups, but a very constant demand for different products due to the minimized usage rate. Heuristics 3–5 are attempts to address both setups and usage. Heuristic 3 is a multiobjective approach striving to attain sequences when minimizing required setups and usage rate are of equal importance. Heuristic 4 also addresses both minimization of required setups and usage rate when more emphasis is placed on setups, while Heuristic 5 places more emphasis on minimization of usage rate. From inspection of table 4a–c, it is clear that these Tabu Search heuristics do in fact succeed in addressing the goals they were designed for. While all three do address both objectives, Heuristic 4

Table 4a. Means of performance measures for Problem Set 1.

| | Heuristic 1 | Heuristic 2 | Heuristic 3 | Heuristic 4 | Heuristic 5 |
|---|---|---|---|---|---|
| Makespan | 35.73 (0.50) | 39.11 (1.6) | 36.64 (0.63) | 36.24 (0.54) | 37.15 (0.82) |
| Setup requirement | 0.23 (0.06) | 0.78 (0.31) | 0.36 (0.12) | 0.30 (0.09) | 0.45 (0.16) |
| Usage rate | 155.4 (86) | 11.08 (4.2) | 37.34 (17) | 67.74 (42) | 22.73 (9.6) |

Table 4b. Means of performance measures for Problem Set 2.

| | Heuristic 1 | Heuristic 2 | Heuristic 3 | Heuristic 4 | Heuristic 5 |
|---|---|---|---|---|---|
| Makespan | 36.55 (0.60) | 39.23 (1.4) | 37.35 (0.85) | 37.14 (0.86) | 37.68 (0.88) |
| Setup requirement | 0.46 (0.14) | 0.89 (0.28) | 0.57 (0.18) | 0.54 (0.17) | 0.64 (0.21) |
| Usage rate | 161.7 (60) | 25.1 (8.7) | 43.8 (20.7) | 52.96 (29) | 32.6 (11.2) |

Table 4c. Means of performance measures for Problem Set 3.

| | Heuristic 1 | Heuristic 2 | Heuristic 3 | Heuristic 4 | Heuristic 5 |
|---|---|---|---|---|---|
| Makespan | 168 (3.65) | 193 (7.2) | 183 (3.4) | 179 (2.4) | 188 (5.6) |
| Setup requirement | 0.14 (0.04) | 0.90 (0.30) | 0.56 (0.21) | 0.44 (0.16) | 0.75 (0.29) |
| Usage rate | 17 949 (7 763) | 172 (59) | 329 (114) | 538 (201) | 204 (78) |

performs better than Heuristic 5 with regard to setups and worse with regard to usage.

It should become quite clear then, that there is an inverse relationship between the required number of setups and usage rate ($F = 400.97$, $t = -20.03$, $P < 0.0001$), which implies that there is a tradeoff between these two performance attributes—it is impossible to simultaneously attain optimal levels of both. As a result, the decision-maker needs to implement a sequence which results in tolerable levels of both required setups and material usage rates for their specific application.

## 7. Conclusions

A technique has been presented to sequence a group of products for a JIT assembly line when setups are necessary. The technique uses Tabu Search to find a sequence when minimization of both material usage rates and setups are of concern. Three different objective functions using the Tabu Search technique are presented that give management the opportunity to address this sequencing problem for varying levels of importance for these two objectives. Providing the user with the ability to place varying levels of emphasis on the two different objectives is important due to the fact that there is a strong inverse relationship between these two objectives—if one objective is adequately addressed, the other objective is most likely inadequately addressed. As a result, the user must have a clear idea of the importance of these two opposing goals so that successful implementation of this heuristic can occur.

## References

Bland, J. A., and Dawson, G. P., 1991, Tabu Search and Design Optimization. *Computer-aided Design*, **23**, 195–201.

Ding, F., and Cheng, L., 1993, An effective mixed-model assembly line sequencing heuristic for just-in-time production systems. *Journal of Operations Management*, **11**, 45–50.

Glover, F., 1990, Tabu Search: a tutorial. *Interfaces*, **20**, 74–94.

Glover, F., 1993, A user's guide to Tabu Search. *Annals of Operations Research*, **41**, 3–28.

Miltenburg, J., 1989, Level schedules for mixed-model assembly lines in just-in-time production systems. *Management Science*, **35**, 192–207.

Monden, Y., 1983, *Toyota Production System* (Norcross, GA: The Institute of Industrial Engineers).

Sumichrast, R. T., and Russell, R. S., 1990, Evaluating mixed-model assembly line sequencing heuristics for just-in-time production systems. *Journal of Operations Management*, **9**, 371–390.

Wantuck, K. A., 1989, *Just-in-Time for America* (Southfield, MI: KWA Media).

## Appendix. Problem sets used for analysis

Problem Set 1:

| Problem | Product 1 | Product 2 | Product 3 | Product 4 | Product 5 |
|---|---|---|---|---|---|
| A | 20 | 0 | 0 | 0 | 0 |
| B | 16 | 1 | 1 | 1 | 1 |
| C | 15 | 2 | 1 | 1 | 1 |
| D | 13 | 4 | 1 | 1 | 1 |
| E | 10 | 5 | 2 | 2 | 1 |
| F | 8 | 7 | 2 | 2 | 1 |
| G | 6 | 6 | 5 | 2 | 1 |
| H | 5 | 5 | 5 | 3 | 2 |
| I | 5 | 4 | 4 | 4 | 3 |
| J | 4 | 4 | 4 | 4 | 4 |

Problem Set 2:

| Problem | Pr 1 | Pr 2 | Pr 3 | Pr 4 | Pr 5 | Pr 6 | Pr 7 | Pr 8 | Pr 9 | Pr 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| A | 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | 11 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| C | 10 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| D | 9 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| E | 8 | 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| F | 7 | 5 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G | 6 | 5 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| H | 5 | 5 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| I | 4 | 4 | 4 | 2 | 1 | 1 | 1 | 1 | 1 | 1 |
| J | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

Problem Set 3:

| Problem | Pr 1 | Pr 2 | Pr 3 | Pr 4 | Pr 5 | Pr 6 | Pr 7 | Pr 8 | Pr 9 | Pr 10 | Pr 11 | Pr 12 | Pr 13 | Pr 14 | Pr 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | 40 | 40 | 8 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| C | 35 | 35 | 10 | 5 | 5 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| D | 30 | 30 | 15 | 10 | 5 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| E | 25 | 25 | 20 | 15 | 5 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| F | 20 | 20 | 20 | 15 | 15 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G | 20 | 20 | 15 | 15 | 10 | 6 | 6 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| H | 15 | 15 | 15 | 10 | 10 | 10 | 10 | 5 | 4 | 1 | 1 | 1 | 1 | 1 | 1 |
| I | 15 | 15 | 10 | 10 | 10 | 10 | 10 | 10 | 4 | 1 | 1 | 1 | 1 | 1 | 1 |
| J | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 6 | 6 | 6 | 6 | 6 |

Note: In the body of each table, the number is simply the demand for that particular product for that particular problem.