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# Multi-objective assembly line balancing via a modified ant colony optimization technique

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A technique derived from ant colony optimization is presented that addresses multiple objectives associated with the general assembly line-balancing problem. The specific objectives addressed are crew size, system utilization, the probability of jobs being completed within a certain time frame and system design costs. These objectives are addressed simultaneously, and the obtained results are compared with those obtained from single-objective approaches. Comparison shows the relative superiority of the multi-objective approach in terms of both overall performance and the richness of information.

*Keywords*: Assembly line balancing; Heuristics; Ant-colony optimization; Efficient frontier

## 1. Introduction

### 1.1 *General assembly line-balancing problem*

Assembly line balancing has been a topic of interest since the 1950s. For comprehensive reviews of assembly line-balancing research, see Becker and Scholl (2003) and Scholl and Becker (2003). The earliest forms of the presented problem, along with the more modern research efforts, have typically concentrated on the minimization of workers needed to staff a line while adhering to task precedence and cycle time restrictions. Mathematically, this objective can be stated as follows:

$$\min \sum_{j=1}^{R} \dot{w}_j \tag{1}$$

$$\text{subject to } t_j \leq C\dot{w}_j, \quad \text{for } j = 1, \ldots, R, \tag{2}$$

where $R$ is the number of work centres, $\dot{w}$ is the (integer-adjusted) number of required workers in work centre $j$, $t_j$ is the estimated time required to complete the tasks in work centre $j$, and $C$ is the prespecified cycle time. In short, with the

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traditional assembly line-balancing problem, it is desirable to place workers in work centres in such a way that as few workers as possible as used, while simultaneously adhering to the policy that no single worker can be 'overloaded'.

Addressing the above problem has been done with success via a variety of ways, from the direct application of simple task selection rules to modern search heuristics. More recently, objectives other than minimization of required workers have been considered. These other objectives typically include system utilization (Askin and Zhou 1997, Gocken and Erel 1998, Vilarinho and Simaria 2002), the probability of jobs being completed within a desired time frame (Merengo *et al.* 1999), system design cost (Askin and Zhou 1997, Rekiek *et al.* 2000, Bukchin and Rubinovitz 2002), the 'evenness' of workload assignments (Ponnambalam *et al.* 2000), etc. Simultaneously addressing these objectives has seen relatively little treatment in the literature. Askin and Zhou (1997), Gocken and Erel (1997), Merengo *et al.* (1999), Ponnambalam *et al.* (2000), Vilarinho and Simaria (2002) and Bukchin and Rubinovitz (2002) have all simultaneously addressed some of the objectives mentioned above, but not all of them. This fact is a primary motivation for the research presented here.

Furthermore, real-world, complicated features of the assembly line-balancing problem have also been considered, such as stochastic task durations and mixed-model assembly (simultaneous assembly of several different items). These complicating features are also addressed here.

### 1.2 *Multiple objectives and the assembly line-balancing problem*

One very compelling reason why few researchers have addressed the multiple objectives of the assembly line-balancing problem simultaneously is because the job is very difficult. Past research (McMullen and Frazier 1998) has indicated that many of these important objectives are in conflict with each other. Frequently, these objectives are directly opposed to each other. For example, past research efforts have suggested that when one obtains a line-balancing solution requiring relatively few workers, there is an associated low probability that these jobs will be completed (assembled) within a certain period. The opposite is also true. When a solution is obtained requiring a relatively large number of workers, there is a high probability that these jobs will be assembled within a certain period. These 'trade-offs' make the simultaneous optimization of multiple objectives a challenge.

Because of this, other measures must be taken. This research looks specifically at two of these. First, a composite function is constructed, with the intent to optimize this function. This function is a linear combination of multiple objectives associated with the line-balancing problem, such as required crew size, system utilization, the probability of jobs being assembled within a certain time frame, and system design cost. The value of this function is intended to serve as an overall measure of the assembly line solution's relative desirability. The second measure taken to address the multiple objectives associated with this problem simultaneously involves the use of an efficient frontier. An efficient frontier, popular in economic analysis and data envelopment analysis, is used to show trade-offs between two or more sometimes conflicting entities. For the research presented here, one of the entities will be the required crew size of an assembly line-balancing solution, while the other entity will be the value of the linear function previously described. Using an efficient frontier in this fashion enables a decision-maker to find the most desired assembly line-balancing solution for each particular crew size.

## 1.3 *Ant colony optimization as a search technique*

Construction of the efficient frontier for a problem such as this cannot be obtained by direct application of a simple rule — problems such as this are termed combinatorial due to their enormous search spaces. Subsequently, one would be well advised to use a modern search heuristic which strives to find global optima. Heuristics well suited for this are simulated annealing (Kirkpatrick *et al.* 1983), tabu search (Glover 1990), genetic algorithms (Goldberg 1989) and ant colony optimization (Dorigo and Gambardella 1997). The present research here adopts a modified version of the ant colony optimization approach.

Ant colony optimization simulates the behaviour of ants, which are social insects that through a form of implied communication can efficiently achieve objectives such as gathering food and assigning tasks to individual agents. The travelling salesman problem (TSP) is a simple way to explain this concept. An ant's job of gathering food can be thought of as an application of the TSP. An ant needs to find food and needs to return to its nest in the most efficient way possible. When ants travel, they leave a pheromone trail (a hormone deposited by the ant). The pheromone attracts other ants. This suggests that a path with a large amount of pheromone will likely attract other ants (pheromone levels dictate the probability of certain links of the network being selected). However, there is also a small probability that a less popular path with also be chosen by ants — this less popular path could eventually lead to a path that is more efficient than the path previously thought to be most efficient. After a modeller simulates several ants traversing the system, the most efficient traversal through the system is noted as the best solution found.

The logic described above to address the TSP can also be adapted to distribute work among assembly line workers. Ants can traverse a list of $n$ jobs requiring placement in an assembly line system. Assuming that task precedence restrictions have been satisfied, an ant could be attracted to tasks eligible for assignment based upon some probabilistic measure of desirability in terms of an objective function value. This probabilistic measure can be thought of as a surrogate measure for pheromone associated with the eligible task. Once the ant has 'visited' (selected) all $n$ tasks, the assembly line solution is complete and its performance measures are noted. The decision-maker repeats this process several times, and the 'best' assembly line solutions in terms of objective function value are reported. In other words, an 'army' of ants is used to find quality solutions to the problem, and the 'best' solution found is noted.

The paper is structured as follows. (1) It presents a methodology based on ant colony optimization techniques to obtain assembly line-balancing solutions for multiple objectives via a composite function and efficient frontier, which is an extension of previous research where only single-objective functions were addressed (McMullen and Tarasewich 2002). (2) It presents a set of test problems. (3) It provides a comparison between the presented multiple objective approach and single-objective approaches. (4) It provides observations and conclusions concerning the methodology.

## 2. Ant colony optimization methodology

Before presenting the steps of the ant colony optimization heuristic, the following definitions are provided:

*Parameters*:

| | |
|---|---|
| $n$ | total number of tasks for problem, |
| $t_i^*$ | expected duration of task $i$, |
| $\hat{\sigma}_i^*$ | estimated standard deviation of task $i$, |
| $C$ | prespecified cycle time, |
| $\alpha h$ | multipliers for objective function ($h = 1, \ldots, 4$), |
| $\alpha$ | work centre creation factor ($0 < \alpha < 1$). |

*Variables*:

| | |
|---|---|
| $L$ | list of tasks for assignment into work centres. |
| $n_j$ | number of tasks in work centre $j$, |
| $R$ | total number of work centres from the solution, |
| $t_j$ | expected duration of all tasks in work centre $j$, |
| $\hat{\sigma}_j$ | estimated standard deviation of work centre $j$, |
| $w_j$ | workers required in work centre $j$, |
| $\dot{w}_j$ | integer-adjusted workers required in work centre $j$, |
| $p_j$ | probability of on-time completion in work centre $j$, |
| $u_j$ | utilization of work centre $j$, |
| $\text{metric}_i$ | evaluation metric associated with task $i$, |
| $ph_i$ | pheromone associated with task $i$, |
| $\mathbf{M}(i, g_i)$ | $n$ by $n$ linkage matrix to used to detail the number of times task $i$ is preceded by task $g_i$. |

Note that $i$ represents a task, while $j$ represents a work centre. In addition, parameters are exogenous, while variables are endogenous.

### 2.1 *Selection of tasks for work centres*

All relevant entities in the above list are initialized to their appropriate values. Before actually selecting a task for membership in the current (non-empty) work centre, a decision must be made whether or not to create a new work centre. This is done via the following relationship:

$$P(\text{new work centre}) = \frac{\alpha}{n_j}, \tag{3}$$

where $j$ is the current work centre. The above relationship guards against a very large number or a very small number of work centres, thereby guarding against high fixed costs (several machines) and high variable costs (several workers). When a new work centre is opened, $t_j$ and $\hat{\sigma}_j$ for new work centre $j$ are initialized to zero.

### 2.2 *Task selection*

In the event of an empty work centre, all relevant statistics are initialized to zero. For each task eligible for membership in $L$, the utilization and probability of on-time completion are calculated to reflect work centre utilization ($u_j$) and probability ($p_j$) if task $i$ were to be added to the current work centre $j$:

$$u_j = \frac{w_j}{\dot{w}_j} \tag{4}$$

$$\text{where } w_j = \frac{(t_j + t_i^*)}{C}, \quad \text{for } i \in L \tag{5}$$

$$\text{and } \dot{w}_j = 1 + \mathrm{int}(w_j) \tag{6}$$

$$p_i = 1\sqrt{2\pi}\int_{-\infty}^{y} \exp(-0.5z^2)\,\mathrm{d}z \tag{7}$$

$$\text{where } Y = \frac{(C[\dot{w}_j - w_j])}{\hat{\sigma}_j}, \tag{8}$$

$$\text{and } \hat{\sigma}_j = \sqrt{\hat{\sigma}_i^2 + \hat{\sigma}_i^{*2}}. \tag{9}$$

Utilization ($u_j$) is a representation of how 'busy' is work centre $j$, while probability ($p_j$) is the work centre's ability to finish its tasks within the cycle time — $p_j$ is determined using numerical integration with 500 subintervals. A busy system typically reflects a low probability of on-time completion, and vice versa. Hence, there is a trade-off between $u_j$ and $p_j$. After determination of $u_j$ and $p_j$, the following multiple-objective function value is determined:

$$\mathrm{metric}_i = a_1u_j + a_2p_j + a_3(u_jp_j) + a_4u_j(1 - p_j). \tag{10}$$

This value, $\mathrm{metric}_i$, is intended to show the relative desirability of adding task $i$ to work centre $j$. It is desired to maximize this value. The first component of this measure provides the utilization contribution. The second component shows the probability of on-time completion contribution. The third component shows the contribution of a composite measure of $u_j$ and $p_j$. The fourth component is included as a surrogate for system design cost — a combination of personnel requirements and equipment requirements. McMullen and Frazier (1998) showed that high probabilities of on-time completion are directly related to large equipment needs, which is the reason for the $(1-p_j)$ term. The $a_h$ terms are user chosen and are intended to be on the [0, 1] interval.

The $\mathrm{metric}_i$ value can then be used to determine the level of pheromone associated with task $i$'s overall attractiveness to enter work centre $j$. The level of pheromone associated with task $i$ is as follows:

$$ph_i = \left(metric_i \Big/ \sum\nolimits_{\mathrm{i}=1}^{n} metric_i\right) + \left(\mathbf{M}(i,g_i) \Big/ \sum\nolimits_{\mathrm{i}=1}^{n} (i,g_i)\right), \quad \text{for all } i \in L. \tag{11}$$

The first part of this relationship is simple to understand: the desirability of task $i$ to enter work centre $j$ relative to all other candidate tasks. The second part relates to the traditional pheromone concept and requires more explanation. The $\mathbf{M}(i,g_i)$ matrix is a representation of historical precedence relationships — it keeps a count of the number of times that task $g_i$ precedes task $i$. Each time that task $g_i$ precedes task $i$, $\mathbf{M}(i,g_i)$ is incremented by 1. The values of $\mathbf{M}(i,g_i)$ are only initialized to zero before simulating the very first ant. These values are not initialized before performing an actual line balance. If task $g_i$ is a frequent predecessor of task $i$ across line balances, then the matrix will reflect this via a high value of $\mathbf{M}(i,g_i)$.

This thinking is intended to resemble the popularity of certain linkages for traversal when ants are simulated to solve the TSP.

Monte Carlo simulation is used to select the task to enter work centre $j$. The relative probability of a task's chance for selection is directly proportional to $\mathrm{ph}_i$. Each time a task is selected for entry into work centre $j$, all relevant statistics are updated to reflect the addition of task $i$ ($u_j, p_j, t_j$ and $\hat{\sigma}_j$) (equations 4–9).

All of the above task selection steps are repeated until all $n$ tasks have been assigned to work centres.

### 2.3 *Determine line balance statistics and construct efficient frontier*

The following is a list of definitions for entities associated with final assembly line-balancing solution:

| | |
|---|---|
| $W$ | number of workers required for the solution, |
| $U$ | utilization of assembly line layout, |
| $P$ | probability of all work centres completing work on time, |
| Cost | design cost of assembly line layout, |
| $S[W]$ | composite objective function value associated with $W$ workers. |

The number of workers required for the recently completed assembly line-balancing solution is as follows:

$$W = \sum_{j=1}^{R} \dot{w}_j. \tag{12}$$

The utilization associated with this solution is as follows:

$$U = \frac{\sum_{i=1}^{n} t_i^*}{CW}. \tag{13}$$

The probability of completing all tasks within cycle time is as follows:

$$P = \prod_{j=1}^{R} p_j. \tag{14}$$

The design cost associated with the assembly line-balancing solution is as follows:

$$\text{Cost} = 30\,000W + 3000 \sum_{i=1}^{R} n_j w_i. \tag{15}$$

The design cost expressed above considers the total cost associated with both personnel and equipment needed to process jobs passing through the assembly line. The major assumptions of this model are that the annual labour cost for an employee is US$30 000/year, and the annual cost for a piece of equipment is US$3000/year. The labour cost can be modified to reflect the actual average cost of employees on the assembly line. In addition, equipment costs might vary according to the tasks performed, the age of the equipment, and which tasks are assigned to a particular workstation. While this model uses fairly simplistic cost assumptions, see Becker and Scholl (2003) for a discussion of research into assembly line equipment cost and selection.

With the individual assembly line-balancing statistics calculated, the multiple-objective measure of performance associated with $W$ workers is as follows:

$$S[W] = a_1 U + a_2 P + a_3 UP + a_4 \left( \left( \underline{Cost} - Cost \right) / \underline{Cost} \right). \tag{16}$$

The above function contains the $a_h$ values as shown in equation (10), and these $a_h$ values are contained in the [0, 1] interval. $\underline{\text{Cost}}$ is the highest possible system design cost for the problem at hand. The values of $a_h$ and their multipliers are all contained on the [0, 1] interval. These objective function multipliers (the $a_h$ values) are generalized here, and are intended to give the manager flexibility emphasize the relationship between $U$, $P$ and design cost, which is most important to their organization. The conclusions provide further elaboration.

The above calculations represented by equations (12)–(16) are performed each time an assembly line-balancing solution is completed. For each solution, the largest value of $S[W]$ is noted for each value of $W$.

The steps above are repeated *ants* number of times — a user-specified number of solutions. The $S[W]$ values and the corresponding values of $W$ then comprise the multiple-objective efficient frontier.

### 2.4 *Numerical example*

To illustrate the use of the presented methodology, a numerical example is provided. The specific example is a 45-task assembly line-balancing problem from the literature (McMullen and Frazier 1998). The cycle time, $C$, was specified to be 10 min. Task durations were randomly generated and their associated standard deviations were assumed to be 15% of the task's duration. Furthermore, task durations and their standard deviations were estimated from a mixed-model problem, where there were assumed to be two different items requiring assembly, where the relative demand was two units of item 'A' demanded for each unit of item 'B'. Therefore, estimates of task duration and its standard deviation are composite estimates. This particular problem caveat recognizes the real-world issues of mixed model assembly, and is discussed in the appendix.

For this example problem, 500 ants were simulated, resulting in 500 assembly line-balancing solutions. Regarding problem parameters specific to the ant colony optimization approach described above, the new work centre factor was set to 0.5, and the $a_h$ values all took on values of 1. Because of this, the maximum possible value for $S[W]$ is 4, and its minimum possible value is zero. Because all $a_h$ values were set equal to 1, all four of the objectives were treated as equally important.

Figure 1 shows the scatter plot of the multiple-objective function value ($S[W]$) and the associated number of workers ($W$). For each number of workers, $W$, it is desired to obtain as high a value as possible for $S[W]$. This suggests desirable levels of utilization, probability of on-time completion and a tolerable design cost. Because it is desired to maximize $S[W]$ for each level of $W$, the relatively inferior values of $S[W]$ can be removed from the plot. Figure 2 shows the result of pruning away the undesired values of $S[W]$. It also shows a line connecting all of the highest values of $S[W]$ for each associated level of $W$. This line can be thought of as the efficient frontier. For the search results of this example problem, this frontier can only be equalled, never surpassed. All points below this frontier are classified as

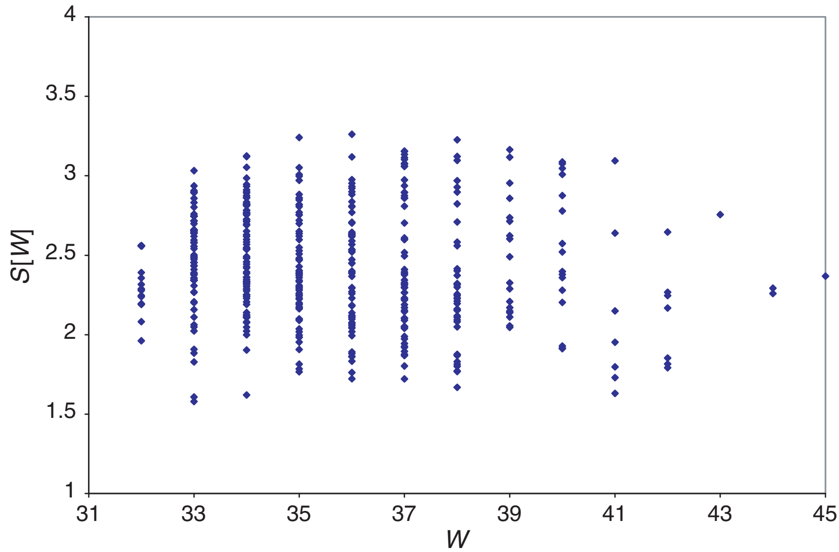


Figure 1. Scatter plot of the composite objective function and the number of required workers for the example problem.

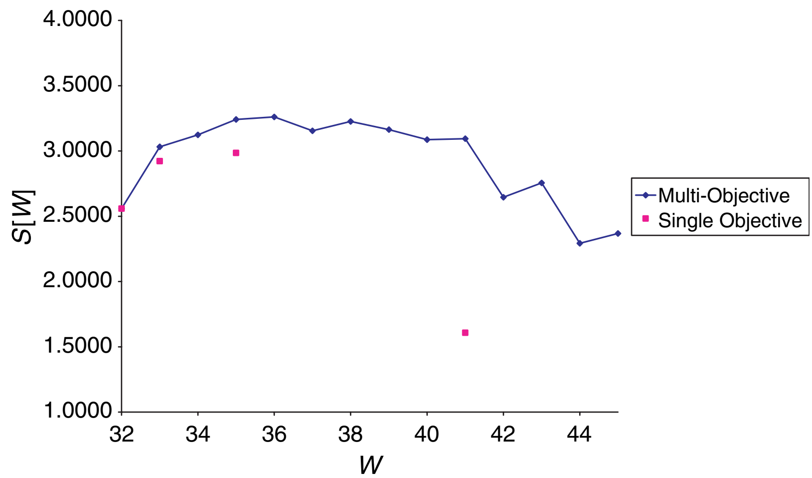


Figure 2. Efficient frontier for the example problem.

relatively inferior. Also included in figure 2 are four additional points. These are the result of solutions obtained using the same search technique presented here, but with single-objectives measured with the multiple objective function in equation (16). Notice how three of these points are in the region classified as relatively inferior (one is 'on' the frontier — associated with $W = 32$ workers).

Table 1 shows the detail of how the $S[W]$ values for each associated value of $W$ on the frontier were determined. An important observation to make regarding table 1 is associated with the values of the entities used to determine $S[W]$ ($U$, $P$ and Cost).

Table 1. Efficient frontier for the example problem.

| $W$ | $U$ | $P$ | $UP$ | Cost | $S[W]$ |
|---|---|---|---|---|---|
| 32 | 0.9853 | 0.7924 | 0.7808 | 5 280 000 | 2.5585 |
| 33 | 0.9554 | 0.8632 | 0.8247 | 3 228 000 | 3.0320 |
| 34 | 0.9273 | 0.9316 | 0.8639 | 3 162 000 | 3.1240 |
| 35 | 0.9008 | 0.9749 | 0.8782 | 2 706 000 | 3.2415 |
| 36 | 0.8758 | 0.9873 | 0.8647 | 2 463 000 | 3.2613 |
| 37 | 0.8521 | 0.9437 | 0.8041 | 2 349 000 | 3.1551 |
| 38 | 0.8297 | 0.9741 | 0.8082 | 2 037 000 | 3.2262 |
| 39 | 0.8084 | 0.9966 | 0.8057 | 2 358 000 | 3.1641 |
| 40 | 0.7882 | 0.9307 | 0.7336 | 1 929 000 | 3.0871 |
| 41 | 0.769 | 0.9479 | 0.729 | 1 854 000 | 3.0947 |
| 42 | 0.7507 | 0.7582 | 0.5692 | 2 283 000 | 2.6458 |
| 43 | 0.7332 | 0.8072 | 0.5919 | 1 989 000 | 2.7557 |
| 44 | 0.7166 | 0.5233 | 0.375 | 1 701 000 | 2.2927 |
| 45 | 0.7006 | 0.5811 | 0.4071 | 1 692 000 | 2.3684 |

Table 2. Details of a single-objective solutions for the example problem.

| Objective | $W$ | $U$ | $P$ | $UP$ | Cost | $S[W]$ |
|---|---|---|---|---|---|---|
| Utilization | 32 | **0.9853** | 0.7924 | 0.7808 | 5 280 000 | 2.5585 |
| Probability | 35 | 0.9008 | **0.9742** | 0.8776 | 4 053 000 | 2.9850 |
| Composite | 33 | 0.9554 | 0.9589 | **0.9162** | 4 797 000 | 2.9220 |
| Cost | 41 | 0.769 | 0.0695 | 0.0534 | **1 497 000** | 1.6084 |

None of these values performs exceptionally well on any single objective, but collectively they perform quite well, which is why their associated $S[W]$ value lies on the efficient frontier.

Contrary to the above finding, table 2 shows in detail how the $S[W]$ values were determined for the single-objective approaches. Here the values typically perform well with respect to only a single objective (their intended objective, in bold), and marginally on the others. For example, the approach intended to perform well with respect to the composite measure ($UP$), does in fact perform quite well with respect to the value of $UP$, but performs poorly with respect to system design cost, resulting in a value of $S[W]$ that is less than what was obtained for the associated 33 workers for the multiple-objective approach.

Note that the value for $\underline{\text{Cost}}$ was 5 280 000 for this example problem.

## 3. Assessment and experimentation

### 3.1 *Comparison of single- and multiple-objective approaches*

Figure 2 shows that the efficient frontier obtained via the multiple objective ant colony optimization search 'lies above' the solutions found via the single-objective approaches, which is an argument in favour of using the multiple objective approach, but the degree to which it is superior to the single-objective approach

Table 3. Comparison of single- and multiple-objective solutions for the example problem.

| $W$ | $S[W]_M$ | $S[W]_S$ | Superiority (%) |
|---|---|---|---|
| 32 | 2.5585 | 2.5585 | 0.00 |
| 33 | 3.0320 | 2.9220 | 3.76 |
| 35 | 3.2415 | 2.9850 | 8.59 |
| 41 | 3.0947 | 1.6084 | 92.41 |

needs to be formalized. A comparison will be made between the single- and multiple-objective approaches using the $S[W]$ calculation for both approaches as shown in equation (16). Because the $S[W]$ values for the single-objective approach do not exist for all possible values of $W$, comparisons will only be made for cases when $S[W]$ values are available for both single- and multiple-objective approaches. $S[W]_S$ will be used to represent the $S[W]$ value obtained for a single-objective approach, while $S[W]_M$ will be used to represent the $S[W]$ value obtained for a multiple-objective approach. Using this notation, the relative superiority of the multiple-objective approach over the single-objective approaches can be assessed as follows:

$$\text{Superiority} = 100 \times \frac{(S[W]_M - S[W]_S)}{S[W]_S}. \tag{17}$$

Table 3 shows details of determining the superiority of the multiple-objective approach over the single-objective approaches for the presented example problem.

When the four values of superiority are averaged, one can then claim that for this problem, the multiple-objective approach provides a composite function that is 26.19% superior to that of the single-objective approaches. In other words, the efficient frontier associated with the multiple-objective approach is 26.19% above the frontier formed by the single-objective approaches. From table 3, it is obvious that the single-objective approach associated with 41 workers is the impetus for its comparatively poor showing. This solution was obtained when the lone objective was minimization of system design cost (table 2), while design cost was held at a tolerable level, system utilization ($U$) and probability of on-time completion ($P$) suffered.

### 3.2 *Problem sets*

To provide a detailed comparison between the methodology presented here with that of a single-objective approach, several test problems are used for assessment. Table 4 provides details of these test problems.

All these test problems have come from the literature. For this research effort, four of the six test problems (29, 40, 45 and 74 tasks) involve mixed-model assembly. For these problems, composite estimates of task durations and their standard deviations were made with respect to the product mix. These estimates are detailed in the appendix. The example problem above is the same as the 45-task problem as shown in table 4.

The new work centre factor, $\alpha$, was always set to 0.5 for the multiple objective approach. This decision was based on pilot study with the intent of finding the most favourable result. The values of $\alpha$ for the single-objective approach were also

Table 4. Details of the test problem sets.

| Problem size | Different products | Product mix | References |
|---|---|---|---|
| 21 tasks | 1 | 1 | Tonge (1965) |
| 25 tasks | 1 | 1 | McMullen and Frazier (1998) |
| 29 tasks | 2 | 2,1 | Buxey (1974) |
| 40 tasks | 3 | 3,2,1 | McMullen and Frazier (1998) |
| 45 tasks | 2 | 2,1 | Kilbridge and Wester (1962) |
| 74 tasks | 4 | 4,2,1,1 | McMullen and Frazier (1998) |

Table 5. New work centre factors ($\alpha$) for the single objective problem, Cost and the number of simulated ants (Cost determined via experimentation).

| Problem size | $U$ ($\alpha$) | $P$ ($\alpha$) | $UP$ ($\alpha$) | Cost ($\alpha$) | Cost | Ants |
|---|---|---|---|---|---|---|
| 21 tasks | 0.5 | 0.4 | 0.1 | 0.5 | 1 488 000 | 250 |
| 25 tasks | 0.5 | 0.9 | 0.2 | 0.5 | 1 995 000 | 250 |
| 29 tasks | 0.5 | 0.6 | 0.3 | 0.5 | 2 691 000 | 250 |
| 40 tasks | 0.5 | 0.5 | 0.2 | 0.7 | 4 800 000 | 500 |
| 45 tasks | 0.5 | 0.6 | 0.2 | 0.6 | 5 280 000 | 500 |
| 74 tasks | 0.5 | 0.3 | 0.3 | 0.6 | 14 364 000 | 1000 |

determined by a pilot study, but a variety of values were found to show the single-objective function value in its best possible light. Table 5 details these values of $\alpha$ for the single-objective approaches. In equation (15), US$3000 is used as the annual machine cost and US$30 000 is used as the annual labour cost/person. These values can be modified to reflect the researcher's desires, but it is emphasized here that regardless of what are these values, the heuristic is always motivated to find the most desired solution in terms of the objective function value. Note too that all $a_h$ values were set to 1, which gives all four components of the $S[W]$ function equal consideration.

Table 5 also shows the number of simulated ants used to obtain the solutions for comparison. This number of simulated ants is the same for both the single- and multiple-objective approaches. This ensures that a fair comparison can be made between the two techniques.

Note too that it is possible that the four different objectives for the single-objective approach can result in two or more solutions having the same value of $W$ for a single problem set. This did not occur in the example problem (tables 2 and 3), but it is possible nonetheless. In the event that this does happen, the $S[W]$ values will be averaged for each associated value of $W$.

### 3.3 *Research question*

There is a single research question here and it is quite straightforward. For all problem sets, does the multiple objective approach provide results superior to those obtained via the single-objective approaches? This question can also be put into the form of hypotheses, where $\mu_{\text{superiority}}$ is used to represent the parameter mean superiority of the multiple-objective technique:

$$H_0: \mu_{\text{superiority}} = 0$$
$$H_A: \mu_{\text{superiority}} > 0.$$

### 3.4 *Computational experience*

The multi-objective ant colony optimization methodology was written using the C++ programming language on a Microsoft Windows platform with an Intel Pentium II 300 MHz processor. The smallest problem set (21 tasks) required 9 CPU seconds, while the largest problem set (74 tasks) required 274 CPU seconds.

## 4. Experimental results

Table 6 shows details of the efficient frontiers of the six test problems, along with a side-by side comparison of how they perform against $S[W]$ values of the single-objective approaches. Note that there were three occasions where the multiple-objective approach provided a result inferior to the single-objective approach (21-, 40- and 74-task problems, with crew sizes $[W]$ of 20, 42 and 61, respectively). However, these three instances are the exception. The average superiority of the multiple- over the single-objective approach was 18.68%, standard deviation = 30.99%. This average superiority of the multiple objective approach is statistically significant at the $\alpha = 0.05$ level, with a $t$-statistic of 2.78 and an associated $p = 0.0056$.

## 5. Discussion and conclusions

### 5.1 *Overall performance of methodology and future research opportunities*

The research presented here shows that the multiple-objective approach clearly outperforms an approach from an earlier research effort (McMullen and Tarasewich

Table 6a. Comparison of multiple- and single-objective approaches for 21- and 25-task problems.

| $W$ | $S[W]_M$ | $S[W]_S$ | Superiority (%) | $W$ | $S[W]_M$ | $S[W]_S$ | Superiority (%) |
|---|---|---|---|---|---|---|---|
| 21 tasks: | | | | 25 tasks: | | | |
| 16 | 2.7380 | 2.7380 | 0.00 | 19 | 2.6831 | 2.5298 | 5.71 |
| 17 | 3.0260 | | | 20 | 3.0261 | 2.8241 | 6.68 |
| 18 | 2.9752 | 2.0011 | 48.68 | 21 | 3.0991 | | |
| 19 | 2.9111 | | | 22 | 3.0254 | | |
| 20 | 2.9091 | 2.9410 | −1.08 | 23 | 2.9810 | | |
| 21 | 2.6539 | | | 24 | 2.8464 | 1.4978 | 47.38 |
| 22 | 2.7817 | | | 25 | 2.5633 | | |
| 23 | 2.3508 | | | 26 | 2.2621 | | |
| | | | | 27 | 1.5533 | | |

Table 6b. Comparison of multiple- and single-objective approaches for 29- and 40-task problems.

| $W$ | $S[W]_M$ | $S[W]_S$ | Superiority (%) | $W$ | $S[W]_M$ | $S[W]_S$ | Superiority (%) |
|---|---|---|---|---|---|---|---|
| 29 tasks: | | | | 40 tasks: | | | |
| 23 | 2.1755 | 2.1755 | 0.00 | 32 | 2.8091 | 2.3004 | 22.12 |
| 24 | 2.9166 | | | 33 | 3.2086 | 3.1573 | 1.63 |
| 25 | 3.044 | 2.9683 | 2.55 | 34 | 3.2582 | | |
| 26 | 3.1235 | 3.0629 | 1.98 | 35 | 3.3369 | | |
| 27 | 3.1136 | | | 36 | 3.2320 | | |
| 28 | 2.9941 | 1.5987 | 87.28 | 37 | 3.2716 | | |
| 29 | 2.8523 | | | 38 | 3.1835 | | |
| 30 | 2.3488 | | | 39 | 3.1562 | 1.8385 | 71.67 |
| 31 | 2.4190 | | | 40 | 2.9236 | | |
| 32 | 1.7312 | | | 41 | 3.0530 | | |
| | | | | 42 | 2.5270 | 3.1248 | −19.13 |
| | | | | 43 | 2.2599 | | |

Table 6c. Comparison of multiple- and single-objective approaches for 45- and 74-task problems.

| $W$ | $S[W]_M$ | $S[W]_S$ | Superiority (%) | $W$ | $S[W]_M$ | $S[W]_S$ | Superiority (%) |
|---|---|---|---|---|---|---|---|
| 45 tasks: | | | | 74 tasks: | | | |
| 32 | 2.5585 | 2.5585 | 0.00 | 57 | 2.4188 | 2.4188 | 0.00 |
| 33 | 3.0320 | 2.9220 | 3.77 | 58 | 3.0308 | 2.921 | 3.76 |
| 34 | 3.1240 | | | 59 | 3.2155 | | |
| 35 | 3.2415 | 2.9850 | 8.59 | 60 | 3.3403 | | |
| 36 | 3.2613 | | | 61 | 3.2225 | 3.3747 | −4.51 |
| 37 | 3.1551 | | | 62 | 3.2867 | | |
| 38 | 3.2262 | | | 63 | 3.3095 | | |
| 39 | 3.1641 | | | 64 | 3.3296 | | |
| 40 | 3.0871 | | | 65 | 3.1394 | | |
| 41 | 3.0947 | 1.6084 | 92.41 | 66 | 3.0497 | | |
| 42 | 2.6458 | | | 67 | 2.5152 | | |
| 43 | 2.7557 | | | 68 | 2.7057 | | |
| 44 | 2.2927 | | | 69 | 2.2014 | | |
| 45 | 2.3684 | | | 70 | 2.0616 | 1.6504 | 24.92 |

2002) when only a single objective was considered when measured over the multiple objectives, as shown in equation (16). The multiple-objective approach is 18.68% superior to the single-objective approach when both approaches are mapped onto the efficient frontier. However, the single-objective function approaches perform as intended. For example, the maximization of the utilization approach results in maximum utilization, etc. The other objectives not directly related to the objective at hand are at the mercy of the heuristic search — occasionally a positive result occurs. The multiple-objective approach does not key on a single objective, but all of them simultaneously. They typically provide fairly good (but not excellent) results across the individual objectives, but when they are considered simultaneously, the results are typically quite strong.

Another way to assess the quality of the methodology presented here is to simulate production runs using the assembly line-balancing solutions resulting from the above methodology and then analyse the simulation results in terms of 'real-world' production measures such as WIP inventory level, the lateness of completions, etc. However, this would provide a tremendous amount of information and perhaps overwhelm the reader when the intent here is simply to provide a methodology concerned with multiple-objective optimization in terms of system utilization, the probability of completion and design cost. Simulated production runs of layouts obtained from this (or a similar methodology) is only mentioned here as a possible future research endeavour.

### 5.2 *Search heuristics as an optimization tool*

As described in detail throughout, the general methodology of choice was a modification of ant colony optimization. This search heuristic is one of many formidable possibilities. Simulated annealing, tabu search, genetic algorithms and artificial neural networks all have their advantages (Michalewicz and Fogel 2002), but the ant colony optimization technique was chosen for two reasons. First, the approach performs well while offering computational efficiency; and second, it is relatively new (Bonabeau *et al.* 1999), so perhaps the reader can benefit from its application to the ALB problem.

### 5.3 *Efficient frontier as a tool for multiple objective optimization*

The efficient frontier itself warrants additional comments because it also provides advantages. In the context of this research, the efficient frontier provides decision-makers with flexibility because the best value of the multiple-objective function is provided for each possible number of workers — such a quantity of information has value. Consider, for example, a situation where management is required to design an assembly line. The efficient frontier approach as presented here can provide management with the ability to react quickly to several forces. Union issues may imply that a minimum crew size is used — the efficient frontier can provide the 'best' arrangement given this crew size. Workspace constraints may imply that a maximum crew size is used — the efficient frontier could subsequently provide the 'best' arrangement given this crew size, etc. Note that the crew size, $W$, was chosen here to serve as the horizontal axis on the efficient frontier because it will always be an integer, making organization of the frontier quite manageable. Another reason why crew size was chosen as the horizontal axis is because it is a very important aspect of the ALB problem.

The multiple-objective function in equation (16) requires some justification. The $a_h$ values were all chosen to be 1 for this objective function so that all four components were considered equally. These $a_h$ values can be adjusted to reflect the decision-maker's desires. As stated above, multiple-objective optimization has not been frequently addressed for the ALB problem, or for many other problems for that matter, because simultaneous optimization of several objectives is not easy to obtain. As a result, researchers must find creative ways to perform multiple-objective optimization. The composite objective function in equation (16) and the efficient frontier approach is the 'creative' contribution here. Data Envelopment Analysis is a possible way to find solutions that excel on several dimensions, and is mentioned here as an opportunity for future research endeavours.

### 5.4 *Contribution of research*

The field of assembly line balancing has been vigorously researched in recent decades. Recently, innovations addressing some of the more complicating features of the problem have been offered. Some of these innovations include parallel treatment of workers, tasks with stochastic durations, multiple objectives (minimum crew, maximum probability of on-time completion and minimum design cost), and mixed-models for JIT systems. All these complicating features are addressed here, which should be of value to decision-makers needing to design competitive assembly lines. This has benefits for the practitioner.

The complicating features of this problem are also addressed in two unique ways. First, the multiple objectives of the problem are addressed via an efficient frontier, which is a simplistic and graphical way to address two or more objectives simultaneously. Second, assembly line design solutions are obtained via the modern search heuristic of ant colony optimization, which is a new application for this newly evolving metaheuristic. These two unique approaches to this problem are beneficial to the researcher interested in the science of production.

## A.1. Appendix

Because this methodology addresses a mixed-model scenario, task durations and their associated variances for each task are estimated to reflect this problem feature. The following definitions are provided:

$t_i^*$ estimated time to complete task $i$,
$t_{i,h}^*$ estimated time to complete task $i$ for item $h$,
$\hat{\sigma}_i^*$ estimated standard deviation for task $i$,
$\hat{\sigma}_{i,h}^*$ estimated standard deviation for task i for item $h$,
$d_h$ demand for item $h$,
$D_T$ total demand for all items,
$Q$ number of unique items in product mix.

The following equations are used to estimate the mean and standard deviations of each task when the product mix is considered:

$$t_i^* = \sum_{h=1}^{Q} \left(\frac{d_h}{D_T}\right) t_{i,h}^* \qquad \text{(A1)}$$

$$\hat{\sigma}_i^* = 0.15 \cdot \sqrt{\sum_{h=1}^{Q} \left(\frac{d_h}{D_T}\right)^2 \left(\hat{\sigma}_{i,h}^*\right)^2} \qquad \text{(A2)}$$

$$\text{where } D_T = \sum_{h=1}^{Q} d_h. \qquad \text{(A3)}$$

A commonly accepted assumption for assembly line-balancing heuristics is that task durations are independent of each other. This is the assumption here as well.

Note that mixed-model production scenarios frequently do not have all tasks performed for all unique items in the production mix. For example, a unit of item A may have all eleven of the possible tasks performed, while a unit of item B may have only nine of the possible eleven tasks performed. When this type of scenario occurs, the two 'missing' tasks associated with item B have task durations of zero, and composite task durations and variations are determined based on this information.

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