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Sequencing for minimal tooling replacements via a variety of objective functions

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This research presents a technique dedicated to obtaining production sequences resulting in minimal tooling replacements. Having sequences that result in fewer tooling replacements can result in savings owing to a smaller requirement for tooling media, and the reduced labour required to change the tooling. A variety of objective functions were used to assess the 'evenness' of tool wear, as it is assumed here that more evenness in tool wear would extend the life of the tooling. Because of the combinatorial complexity of sequencing problems, simulated annealing was used as a search engine to find sequences having desirable levels of the aforementioned objective function values. These obtained sequences were then simulated in a manufacturing environment to determine how many tooling replacements were required for the objective function of interest. Experimentation has demonstrated that the research presented here provides results generally superior to those of an earlier published effort in terms of the required tooling replacements (McMullen *et al.* 2002).

1. Introduction

Responsible sequencing is an important part of operations management and logistics. When job sequencing is done well, a firm can enhance its competitive position by having smaller in-process inventories, shorter process times, require fewer workers and the like. Conversely, sequencing that is not well thought out can have the opposite effect, which can be damaging to the competitive position of the firm. Most production sequencing research has been dedicated to finding sequences resulting in minimal WIP levels (Ding and Cheng 1993), minimal make-span times and so on.

Very little research, however, has been done that is dedicated to finding production sequences resulting in minimization of tooling replacements. There have been several published efforts concerned with the physical means to increase and/or improve tooling life (Barad and Hoang. 1995, Kee 1994, Leung and Tanchoco 1990, Luxhoj 1992, Yeo 1995), but very little effort concerned with finding actual sequences serving the same purpose. Minimization of tooling replacements is an important objective, especially when expensive tooling media (such as industrial-grade cutting diamonds) and/or lengthy changeover times are involved. The research presented here is dedicated to finding sequences resulting in minimal tooling replacements. The ritual of having the tyres of one's car periodically rotated is a relevant example. Every so often, a car owner has the four tyres on their car rotated. The primary reason this is done is to extend the life of the tyres, subsequently saving the car's owner the expense and hassle of prematurely replacing the tyres. When tyres

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are rotated, the tyre is moved to a different position on the car—this permits tyre wear on a different part of the tyre. Therefore, the tyres wear more evenly than if they were never rotated.

This virtue of tyre rotation can be applied to industrial processing as well. Several industrial applications involve tool wear: textile cutting, carpet cutting, paper cutting and milling, to name just a few. All of these applications could have the life of their tooling extended (and subsequently save the firm money) if careful thought were put into the sequence of jobs to be processed.

One of the primary difficulties associated with finding sequences to minimize the required tooling replacements via evenly distributing tool wear is finding some objective function value that will accurately reflect the 'evenness' of tool wear. The research presented here addresses this challenge by evaluating a variety of objective functions, and determining which of them works best in a simulated production environment, in terms of required tooling replacements. Another difficulty with this type of research is the combinatorial nature of sequencing problems—small increases in the problem-size result in large increases in the number of feasible solutions. In short, finding optimal solutions to all sequencing problems is far from possible. To combat this problem, sequences are obtained via the search heuristic of simulated annealing (using the presented objective functions as the guiding force). Complete enumeration, along with simulated annealing, is used to find optimal solutions (in terms of the presented objective functions) for the smaller test problems.

The sequences obtained for several test problems via enumeration and/or simulated annealing are evaluated for their associated required replacements via a simulated production run. Experimentation is then used to determine the desirability of these sequences obtained via the objective function strategies, and comparisons are made between the methodology presented here and methodology presented in an earlier research effort (McMullen *et al.* 2002).

The following sections of this paper detail the proposed methodology and objective functions; briefly discuss the concept of simulated annealing; outline the experimental effort to compare the desirability of sequences; and offer concluding comments.

2. Measuring tool wear via objective functions

2.1. *Tool wear*

To assist the reader in having a clear understanding of uniformity in tool wear, consider figures 1(a) and (b).

Figure 1(a) shows three sources for tool wear (three regions of a cutting surface, for example) where there has been more wear in Areas I and III than there has been in Area II. Conversely, figure 1(b) shows three sources for tool wear displaying a more even distribution of tool wear.

Assuming that all tooling must be replaced when at least one component of the tooling requires replacement, figure 1(b) shows a much more desirable situation compared with figure 1(a) because the tooling in figure 1(a) will require more frequent replacement than the tooling shown in figure 1(b). If decision-makers can find production sequences that evenly distribute tool wear across surfaces, they will be successful in minimizing the number of required tooling replacements.

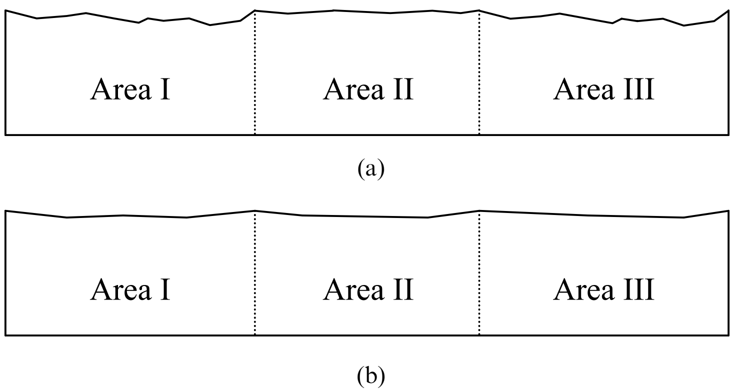

Figure 1. (a) Uneven distribution of tool wear. (b) Even distribution of tool wear.

2.2. *Tool wear matrices*

One of the major challenges one must face to find sequences resulting in a minimal number of tooling replacements is associated with measurement. In other words, how will one know how well a sequence will perform regarding replacements? To address this important question, a variety of objective functions are presented in an attempt to 'quantify' a sequence's ability to minimize required tooling replacements.

Prior to the presentation of the actual objective functions used for this research, the following values are defined:

n the number of items requiring sequencing,
m the number of sources for tool wear,
p the number of unique items in the sequence,
i index for n $(1, 2, \ldots, i, \ldots, n)$,
j index for m $(1, 2, \ldots, j, \ldots, m)$,
k index for p $(1, 2, \ldots, k, \ldots, p)$,
d_k demand for item k,
$\mathbf{W}$ the n by m wear matrix,
w_{ij} the element in the ith row and jth column of $\mathbf{W}$,
$\mathbf{C}$ the n by m cumulative wear matrix,
c_{ij} the element in the ith row and jth column of $\mathbf{C}$,
$\mathbf{R}$ the n by m gradient matrix,
r_{ij} the element in the ith row and jth column of $\mathbf{R}$,
$\max(c_i)$ maximum element in the ith row of $\mathbf{C}$,
$\min(c_i)$ minimum element in the ith row of $\mathbf{C}$,
g_i difference in $\max(c_i)$ and $\min(c_i)$ in the ith row of $\mathbf{C}$,
$\bar{g}$ mean value of g_i for matrix $\mathbf{C}$,
τ upper-threshold value for tool wear,
ω number of sequence replications,
κ number of deterministic replacements via heuristic,
Z_c objective function value of current solution,
Z_v objective function value of variant solution,
Δ percent change in objective function values,
k_b Boltzmann constant,

Item (i)	Item label	w_{i1}	w_{i2}	w_{i3}	w_{i4}
1	A	3	6	2	9
2	A	3	6	2	9
3	B	1	4	3	10
4	B	1	4	3	10
5	C	3	9	7	3
6	C	3	9	7	3
7	D	5	7	6	5
8	D	5	7	6	5
9	E	5	10	9	9
10	E	5	10	9	9

Table 1. Example of tool wear matrix **W**.

T simulated annealing control parameter,
CR simulated annealing cooling rate,
$Tool_j$ tooling remaining for wear source j,
variant degree of variation in tool-wear simulation.

As an example of the tool wear matrix (**W**), consider the following example problem (table 1), involving ten items in the sequence ($n = 10$), and four different sources for tool wear ($m = 4$).

The first column of table 1 is the row index i. The second column is a simple item label—for this example there are five unique items requiring sequencing (A–E). Columns three through six are the w_{ij} values of the **W** matrix. The value for w_{83} for this example is 6. That is, the third column value in the eighth row has a value of 6. It is important to notice how each unique item has tool wear values that are identical to each other. In other words, the tool wear values for each item 'A' will be the same, the tool wear values for each item 'B' will be the same, etc. From a practical standpoint, this should make sense—each item A that is processed through the tooling will wear in exactly the same way, but will wear differently from the other items that are processed through the tooling.

The cumulative tool wear matrix **C** is derived directly from **W**. The individual matrix values for **C** are determined as follows:

$$c_{1j} = w_{1j}, \text{ for } j = 1, \ldots, m, \tag{1}$$

$$c_{ij} = c_{i-1,j} + w_{ij}, \text{ for } i = 2, \ldots, n;\ j = 1, \ldots, m. \tag{2}$$

Table 2 shows the values of the **C** matrix derived from the **W** matrix values from table 1.

2.3. *Objective functions*

Several objective functions are now presented that exploit the tool wear matrix (**W**) and the cumulative tool wear matrix (**C**). All of these objective functions are intended to guide the search process such that sequences are obtained that will subsequently minimize the required tooling replacements.

2.3.1. *Objective function 1: minimal deterministic replacements*

The first presented objective function is intended to minimize directly the number of required changeovers. Mathematically, this is expressed as follows:

$$\text{Min: } \kappa. \tag{3}$$

Item (i)	Item label	c_{i1}	c_{i2}	c_{i3}	c_{i4}
1	A	3	6	2	9
2	A	6	12	4	18
3	B	7	16	7	28
4	B	8	20	10	38
5	C	11	29	17	41
6	C	14	38	24	44
7	D	19	45	30	49
8	D	24	52	36	54
9	E	29	62	45	63
10	E	34	72	54	72

Table 2. Example of tool wear matrix **C**.

For this objective function, the values in the **W** and **C** matrices are treated as deterministic. When the obtained production sequences are simulated, these values of **W** and **C** are treated as stochastic (this will be further elaborated upon on the experimentation section).

For this objective function, the obtained sequence is replicated ω times. Each time a tooling replacement is required, the number of required replacements κ is incremented by one. A tooling replacement is required whenever the maximum c_{ij} value for row i in **C** is greater than or equal to the threshold value τ. Mathematically, this is as follows:

$$\text{if } \max(c_{i,1}, c_{i,2}, \ldots, c_{i,m}) \geq \tau, \text{ then } \kappa = \kappa + 1, \text{ and} \tag{4}$$
$$(c_{i+1,1}, c_{i+1,2}, \ldots, c_{i+1,m}) = (w_{i+1,1}, w_{i+1,2}, \ldots, w_{i+1,m})$$
$$\text{else } \kappa = \kappa, \text{ and} \tag{5}$$
$$(c_{i+1,1}, c_{i+1,2}, \ldots, c_{i+1,m}) = (c_{i,1} + w_{i+1,1}, c_{i,2} + w_{i+1,2}, \ldots, c_{i,m} + w_{i+1,m}).$$

In other words, if the cumulative wear on any of the tooling sources is at least the value of the tooling threshold τ, the replacement counter, κ, is incremented by one, and the corresponding values of **C** for the next item in the sequence take on the $w_{i,j}$ values. Otherwise, no replacements are needed, and the values of the **C** matrix are updated according to equation (2). As an example, consider a simple scenario where the value of τ is 25, and there are two sequence replications ($\omega = 2$) for the example above. In other words, the sequence in table 1 is repeated twice. Table 3 details this example.

Notice from table 3 how the values of the **C** matrix are 'reset' after tooling replacements have been made. The example problem results in six required tool changes. The difference between the application of this objective function and the results of simulated production runs from the sequences is that with the objective function here, the values of **W** are treated as deterministic, while in the simulated production run the values of **W** are treated as stochastic. The above example is obviously a simplified one, and is intended solely for the purpose of illustrating computation of the objective function value.

2.3.2. *Objective function 2: minimum gap sum*

The intent of this approach is to minimize the sum of the gap between the maximum and minimum values of each row in the **C** matrix. Mathematically, this is as follows:

Item (i)	Item label	$w_{i1}(c_{i1})$	$w_{i2}(c_{i2})$	$w_{i3}(c_{i3})$	$w_{i4}(c_{i4})$	$\max(c_i)$	κ
1	A	3 (3)	6 (6)	2 (2)	9 (9)	9	0
2	A	3 (6)	6 (12)	2 (4)	9 (18)	18	0
3	B	1 (7)	4 (16)	3 (7)	10 (28)	28	1
4	B	1 (1)	4 (4)	3 (3)	10 (10)	10	1
5	C	3 (4)	9 (13)	7 (10)	3 (13)	13	1
6	C	3 (7)	9 (22)	7 (17)	3 (16)	22	1
7	D	5 (12)	7 (29)	6 (23)	5 (21)	29	2
8	D	5 (5)	7 (7)	6 (6)	5 (5)	7	2
9	E	5 (10)	10 (17)	9 (15)	9 (14)	17	2
10	E	5 (15)	10 (27)	9 (24)	9 (23)	27	3
1	A	3 (3)	6 (6)	2 (2)	9 (9)	9	3
2	A	3 (6)	6 (12)	2 (4)	9 (18)	18	3
3	B	1 (7)	4 (16)	3 (7)	10 (28)	28	4
4	B	1 (1)	4 (4)	3 (3)	10 (10)	10	4
5	C	3 (4)	9 (13)	7 (10)	3 (13)	13	4
6	C	3 (7)	9 (22)	7 (17)	3 (22)	22	4
7	D	5 (12)	7 (29)	6 (23)	5 (29)	29	5
8	D	5 (5)	7 (7)	6 (6)	5 (7)	7	5
9	E	5 (10)	10 (17)	9 (15)	9 (17)	17	5
10	E	5 (15)	10 (27)	9 (24)	9 (27)	27	6

Table 3. Computation of replacements for example problem with two sequence replications and a tooling threshold of 25.

$$\min : Z = \sum_{i=1}^{n} g_i, \tag{6}$$

$$\text{where } g_i = \max(c_i) - \min(c_i). \tag{7}$$

The rationale here is to find a sequence with consistency in each row of **C**, with the ultimate intent of 'smoothing' the wear of the tooling and, hopefully, having minimal tooling replacements.

2.3.3. *Objective function 3: maximum gap sum*

This objective is intended to maximize the sum of the gaps between the maximum and minimum values of the rows of **C**. Obviously, this is the opposite strategy of the objective previously defined. The intent here is to have 'lumpiness' from row to row of **C**, with the ultimate intent of minimal tooling replacements. Mathematically, the objective is as follows—the value of g_i is defined in equation (7):

$$\max : Z = \sum_{i=1}^{n} g_i. \tag{8}$$

2.3.4. *Objective function 4: minimum gap standard deviation*

The intent of this objective is similar to that of Objective function 2—minimization of gaps in the rows of **C**. Here, however, a standard deviation metric is used instead of a gap total—more of a standardized gap measure:

$$\min : Z = \sqrt{\sum_{i=1}^{n} (g_i - \bar{g})^2 / (n-1)}, \qquad (9)$$

where

$$\bar{g} = \sum_{i=1}^{n} g_i / n. \qquad (10)$$

2.3.5. *Objective function 5: maximum gap standard deviation*

The intent of this objective is again the opposite of the objective presented previously. Here it is desired to maximize the standard deviation of the gaps of the rows in **C**. This is an attempt to maximize the 'lumpiness' of row-by-row gaps, with the hope of reducing the required tooling replacements. Mathematically, this objective is stated as follows:

$$\max : Z = \sqrt{\sum_{i=1}^{n} (g_i - \bar{g})^2 / (n-1)}. \qquad (11)$$

2.3.6. *Objective function 6: minimize gradient measure*

This objective function is intended to minimize the average amount of percentage increase in a cumulative wear matrix from one item in the sequence to the next. This objective function is expressed as follows:

$$\min : Z = \sum_{i=2}^{n} \sum_{j=1}^{m} r_{ij} / ((n-1)^* m), \qquad (12)$$

where

$$r_{ij} = 100^* (c_{i,j} - c_{i-1,j}) / c_{i-1,j}. \qquad (13)$$

This objective is similar in spirit to those stated in equations (6) and (9)—an attempt to 'smooth' the cumulative wear from one item to the next.

2.3.7. *Objective function 7: maximize gradient measure*

This objective function is the maximization version of the one just presented. Mathematically, it is expressed as follows:

$$\max : Z = \sum_{i=2}^{n} \sum_{j=1}^{m} r_{ij} / ((n-1)^* m), \qquad (14)$$

where r_{ij} is determined as shown in equation (13).

2.4. *Objective function example*

Table 4 shows the derivation for the total gap and gap standard deviation measures for the example developed previously, while table 5 shows the derivation of the values of the gradient matrix **R**.

For this example, the total gap measure is 264 (the sum of the gap_i column) and the gap standard deviation measure is 9.5242 (the standard deviation of the gap_i column). The average gradient measure is 35.49% (the average of the values in the **R** matrix).

Item (i)	Item label	c_{i1}	c_{i2}	c_{i3}	c_{i4}	$\min(c_i)$	$\max(c_i)$	gap_i
1	A	3	6	2	9	2	9	7
2	A	6	12	4	18	4	18	14
3	B	7	16	7	28	7	28	21
4	B	8	20	10	38	8	38	30
5	C	11	29	17	41	11	41	30
6	C	14	38	24	44	14	44	30
7	D	19	45	30	49	19	49	30
8	D	24	52	36	54	24	54	30
9	E	29	62	45	63	29	63	34
10	E	34	72	54	72	34	72	38

Table 4. Derivation of gap total and gap standard deviation for example problem.

Item (i)	Item label	r_{i1}	R_{i2}	r_{i3}	r_{i4}
1	A	N/A	N/A	N/A	N/A
2	A	100.00	100.00	100.00	100.00
3	B	16.67	33.33	75.00	55.56
4	B	14.29	25.00	42.86	35.71
5	C	37.50	45.00	70.00	7.89
6	C	27.27	31.03	41.18	7.32
7	D	35.71	18.42	25.00	11.36
8	D	26.32	15.56	20.00	10.20
9	E	20.83	19.23	25.00	16.67
10	E	17.24	16.13	20.00	14.29

Table 5. Derivation of gradient matrix values for example problem.

It is important to note that the first objective function (minimize deterministic replacements) requires the sequence to be replicated ω times. This objective also requires the decision-maker to decide on the threshold value of τ. The other six objective functions only require a single replication of the sequence, and the **C** matrix essentially determines their values.

3. Finding sequences with desirable objective function values

3.1. *Combinatorial complexity*

The example problem developed above exploited the following sequence: AABBCCDDEE. This is one of many possible sequences. Another possible sequence could be: BCEADDCEAB. This second sequence would result in $\kappa = 5$ deterministic replacements (with parameters of $\omega = 2$ and $\tau = 25$), a total gap measure of 104, a gap standard deviation measure of 5.3375, and a mean gradient of 46.47%. The point of this comparison is to emphasize the fact that each unique sequence could provide differing results of the objective function values presented here, and the primary interest here is to find sequences with the optimal objective function values.

If there are n unique items requiring sequencing, there will be $n!$ possible sequences. If there are multiple units demanded for each of the unique items requiring sequencing, the number of possible sequences is as follows:

$$\text{possible sequences} = \left(\sum_{k=1}^{p} d_k\right)! \Big/ \prod_{k=1}^{p} d_k!. \tag{15}$$

For the example problem above, there are 113 400 possible sequences (10!/32). If there are ten unique items requiring sequencing, there are 3 628 800 (or 10!) possible sequences.

It becomes immediately clear that small increases in the size of the problem result in subsequently large increases in the number of solutions possible. This means that enumerating all of the possible solutions for even small sequencing problems becomes problematic. Problems of this type are frequently referred to as combinatorial optimization problems. Subsequently, the decision-maker must find a way to obtain sequences providing optimal (or near-optimal) results, but with a reasonable degree of computational effort.

3.2. *Search heuristics and simulated annealing*

Simulated annealing is a search heuristic that is widely recognized for its ability to obtain near-optimal results for combinatorial optimization problems with a reasonable amount of computational effort (Kirkpatrick *et al.* 1983, Eglese 1990). Simulated annealing works by using a simple stochastic mechanism to find a variant of a current solution. An example of this is as follows:

$$\text{Current Solution}\quad \text{AA}\underline{\text{B}}\text{BCCD}\underline{\text{D}}\text{EE}$$

$$\text{Variant Solution}\quad \text{AA}\underline{\text{D}}\text{BCCD}\underline{\text{B}}\text{EE}$$

In this example, the underlined elements of the current solution, 'B' and 'D' are swapped, resulting in the variant solution. The underlined elements are randomly selected. After the variant solution is obtained, its objective function value is determined. If the objective function value of the variant solution (Z_v) is superior to that of the current solution's objective function value (Z_c), the variant solution is promoted to the current solution. Otherwise, the following value is determined:

$$\Delta = (Z_v - Z_c)/Z_c. \tag{16}$$

This value of Δ is then used to determine the probability of this relatively inferior solution being accepted (P(A)) as current via the following:

$$\mathrm{P(A)} = \exp(-\Delta/k_b T). \tag{17}$$

For objective functions of the maximization variety, equation (17) undergoes a sign change. The value of the Boltzmann constant (k_b) is user-defined. T has initial and final values that are user-defined. After a certain number of user-defined iterations, the value of T is updated via the following:

$$T = T \times CR. \tag{18}$$

The important feature of simulated annealing is that relatively inferior solutions are sometimes accepted as current solutions in attempt to avoid local optima trapping. The probability of accepting an inferior solution diminishes as the search progresses (Metropolis *et al.*, 1953), as governed by equation (17).

4. Experimentation

Experimentation is performed to determine which of the above objective functions, or strategies, perform desirably in terms of minimal replacements. To determine this, the production sequences obtained via the presented objective functions are simulated in a manufacturing environment, and the number of replacements is then determined. Once the replacements associated with each sequence have been captured, research questions regarding the performance of the objective functions are explored.

4.1. *Simulation of production sequences*

The production simulation is quite straightforward. Each time an item in a sequence is processed, the level of available tooling for each source ($Tool_j$) is adjusted according to the following relationship:

$$Tool_j = Tool_j - (w_{ij} + variant), \text{ for } j = 1, \ldots, , m, \tag{19}$$

where

$$variant = w_{ij} \times cv \times z, \tag{20}$$

$$\text{if } variant \leq -0.2w_{ij}, \text{ then } variant = -0.2w_{ij}. \tag{21}$$

After each item i is processed in the sequence, a check is made to see if the level of available tooling has fallen below the value of zero. If the amount of available tooling has fallen below the value of zero, then the number of replacements (κ) is incremented by one. If a tooling replacement is required, the value of available tooling for all sources is reset to the threshold level (τ). For the experimentation here, the value of the threshold (τ) is 50, and each simulated sequence is replicated 2000 times to capture a long-term measure of replacements. For each unique combination of experimental factors, each sequence simulation run is repeated 25 times so that a reasonable estimate of required replacements is possible.

Additional explanation is required regarding equations (19)–(21). The level of *variant* is the stochastic component of the tool-wear simulation—it is dependent on the coefficient of variation (cv) and the value of the randomly generated normal deviate (z). The cv value gives the researcher general control over the amount of variation in the tool wear simulation—smaller values of cv result in less variation in tool wear, while larger values of cv result in more variation in tool wear. For the research presented here, three different values of cv are used as experimental factors: 0.05, 0.15 and 0.25. The randomly generated normal deviate is provided by the CPU's random number generator, and is defined by the Box–Muller Transformation (Box and Muller, 1953):

$$z = \sqrt{-2\ln(r_1)}\cos(2\pi r_2), \tag{21}$$

where r_1 and r_2 are uniformly distributed random numbers on the $[0, 1]$ interval. Equation (20) is employed to prevent minimal tooling wear—making certain that negligible tool wear does not occur from job to job within the sequence. This is done to account for the fact that some non-negligible degree of tool wear will occur from job to job within a sequence. It is also appropriate to note that no similar 'guards' have been employed to prevent large amounts of tool wear, because it is assumed that a large amount of tool wear can, in fact, occur in practical circumstances.

4.2. *Research questions*

To assess the general performance of the presented methodology, the following research questions are constructed.

(1) Do the 'smoothness' approaches (Objective functions 2, 4 and 6) have results that are preferred to the 'lumpiness' approaches (Objective functions 3, 5 and 7) in terms of tooling replacements?

Problem set	Wear areas (m)	Product-mix	Range of tool-wear	Feasible solutions	Mix var.	CR
1	3	A(1), B(1), C(1), D(1), E(1), F(1), G(1), H(1)	1–3	40 320	0.422 (0.0)	0.8191
2	3	A(1), B(1), C(1), D(1), E(1), F(1), G(1), H(1), I(1)	1–3	362 880	0.316 (0.0)	0.9780
3	3	A(1), B(1), C(1), D(1), E(1), F(1), G(1), H(1), I(1), J(1)	1–7	3 628 800	0.0 (0.0)	0.9970
4	4	A(4), B(3), C(2), D(1), E(1), F(1)	1–8	1 663 200	1.398 (1.265)	0.9936
5	4	A(2), B(2), C(2), D(2), E(2)	1–10	113 400	1.054 (0.0)	0.9315
6	3	A(3), B(3), C(2), D(2), E(2)	1–25	1 663 200	1.317 (0.548)	0.9936
7	5	A(4), B(4), C(3), D(3), E(2), F(2), G(1), H(1)	1–9	$2.993(10)^{13}$	1.491 (1.195)	0.9999
8	3	A(12), B(3), C(2), D(2), E(1)	1–25	219 629 600	3.682 (4.528)	0.9999
9	3	A(12), B(3), C(2), D(2), E(1)	1–15	219 629 600	3.682 (4.528)	0.9999
10	3	A(4), B(4), C(4), D(4), E(4)	1–15	$3.0554(10)^{11}$	2.108 (0.0)	0.9999
11	3	A(4), B(4), C(4), D(4), E(4)	1–30	$3.0554(10)^{11}$	2.108 (0.0)	0.9999

Table 6. Details of problem sets.

(2) Do the objective function approaches presented here result in fewer replacements as compared with the objective function approaches presented by the work of McMullen *et al.* (2002)?
(3) Does the minimal deterministic replacements approach (Objective function 1) have results that are preferred to the other smoothness-oriented approaches (Objective functions 2–7) in terms of tooling replacements?
(4) Of the seven objective functions presented here, which performs best?
(5) Do solutions obtained via complete enumeration provide more desirable results in terms of tooling replacements than those obtained via simulated annealing?
(6) What effect, if any, does the coefficient of variation (cv) have on replacement performance?

These research questions are addressed using univariate ANOVA.

4.3. *Problem sets*

Eleven different problem sets are used to assess the performance of the presented heuristics so that generalized performance observations can be made. Details of these problems are shown in table 6. These problems were extracted from the literature (McMullen *et al.* 2002), and are intended to provide a variety of optimization challenges in terms of problem size and problem variability. In this context, the term ‘variability’ is intended to suggest variability with regard to both range of tool wear and variation in product mix.

The first column is the problem set identifier. The second column shows the number of sources for tool wear (m). The third column shows the actual product-mix. For example, Problem set 5 has a demand of two units each for items A, B, C, D and E. The fourth column shows the range of the w_{ij} values (the degree of tool wear). The fifth column shows the number of feasible solutions for the particular sequence. The sixth column shows the variation of the product-mix. The first entry for each problem in this column is the standard deviation associated with the units of demand for each unique item in the product mix when a maximum of ten possible unique items is considered. The second entry for each problem in this column (shown in parentheses) is the standard deviation associated with the units of demand for each unique item in the product mix when only the number of unique items is considered. For example, Problem set 9 has a product mix standard deviation of 3.682 when ten unique items are considered (12, 3, 2, 2, 1, 0, 0, 0, 0, 0), and a standard deviation of 4.528 when only the five unique items are present (12, 3, 2, 2, 1). The seventh column shows the cooling rate (CR) which guides the simulated annealing process. It should be noted that Problems sets 1 through 6 have their solutions obtained via both complete enumeration and simulated annealing—the larger problem sets have their solutions obtained via simulated annealing only because of their enormity.

Considerable effort was made to have the problem sets exhibit diversity in terms of both computational complexity and variation, to enable the generalized articulation of findings (Rardin and Uzsoy 2001, Hooker, 1995). The smallest problem set had 40 320 possible solutions, while the largest had 3.0554×10^{11} possible solutions. In terms of problem variability, tool wear spans from as little as 1–3 units, and as much as 1–30 units. The product mix varies from as little as no variation (a standard deviation of zero), to a relatively large degree of variation (a standard deviation of 4.528, when only the number of unique items in the mix is considered). It should also be noted that controlled randomization techniques were used to obtain product mixes and tool wear ranges so that diversified problem sets could be used for assessment.

As stated earlier, the fundamental objective of this research is to minimize replacements. One problem in comparing heuristics is the effect of the problem size and the range of tool wear—problems with a larger range of tool wear will result in more replacements. This causes difficulty in making comparisons across problem sets. To compensate for this, a ratio of replacements to minimum replacement for each problem set is constructed. Consider, for example, three simulated results for a sequence that results in the following number of replacements: 1008, 1000 and 1012. From this group, this minimum number of replacements is 1000. The ratios would then be 1.008 (1008/1000), 1.000 (1000/1000) and 1.012 (1012/1000), respectively. Consider another set of simulated results for a larger problem set yielding the following number of replacements: 5032, 5018 and 5025. From this group, the minimum number of replacements is 5018. The ratios associated with these three results would be 1.0028 (5032/5018), 1.0000 (5018/5018) and 1.0014 (5025/5018) respectively. When this type of transformation takes place, these ratios can then permit the researcher to make comparisons across all problem sets.

It is also appropriate to note that the simulated annealing parameters for the previously published heuristics are identical to those presented here. This is desired so that a fair comparison can be made between the earlier research effort and the research presented here.

Factor	Explanation	Levels
Objective function:	Five SA-based objective functions from earlier research effort used for all 11 problems	55
	Three enumeration-based objective functions from earlier research effort used for Problems 1–6	18
	Seven SA-based objective functions from current research effort used for all 11 problems	77
	Three enumeration-based objective functions from current research effort used for Problems 1–6	42
Coefficient of variation (*cv*)	Three levels of *cv*: 0.05, 0.15 and 0.25	3

Table 7. Detail of experimental factors.

4.4. *Experimental factors*

There are several experimental factors associated with this research effort. Table 7 details these factors.

From inspection of table 7, one can see that there are 192 $(55 + 18 + 77 + 42)$ unique combinations. Each of these combinations is subjected to 25 replications of the tool wear simulation for each of the three levels of the coefficient of variation (*cv*) factor. This results in a database of 14 400 observations $(192 \times 25 \times 3)$.

4.5. *Comparison with performance from earlier research*

One of the research questions (Research question 2) is concerned with comparing the performance of the methodology presented here with that of an earlier research effort (McMullen *et al.* 2002). This earlier research effort constructed production sequences by measuring a sequence's correlation between adjacent members of the actual sequence, via the tool wear matrix of each unique item in the sequence. It was desired to obtain sequences such that the aggregate correlation between adjacent members of the sequence was minimized.

5. Experimental results

5.1. *Research question 1*

Regarding the first research question, the minimization approaches (Objective functions 2, 4 and 6) are more effective than the maximization approaches (Objective functions 3, 5 and 7)—the difference is significant at the $\alpha = 0.05$ level ($F = 3.84$, $p = 0.05$). The mean ratio for the minimization approaches is 1.0277, while the mean ratio for the maximization approaches is 1.0287. Table 8 shows a further breakdown of this comparison, detailed by the three strategies of concern here—total gap, standard deviation of gap, and gradient.

Table 8 shows that the minimization strategies are preferred to the maximization strategies. The differences are not overwhelming, but are nevertheless consistent, and support the overall claim that the minimization approach is preferred to the maximization approach.

Because of the relative desirability of the minimization approaches, the general intent of the minimization strategies will be considered hereafter—they induce 'smoothness' in the cumulative tool wear matrix, which result in slightly better sequences than those obtained via the maximization strategies. Subsequently, all research questions hereafter will deal with analyses from the data set that includes

Strategy	Minimization ratio	Maximization ratio	*F*-statistic	*p*-value
Gap total	1.0279	1.0280	0.01	0.91
Gap standard deviation	1.0287	1.0300	2.01	0.16
Gradient	1.0264	1.0284	3.52	0.06

Table 8. Comparison of minimization and maximization strategies.

sequences obtained from the minimization approaches (Objective functions 2, 4 and 6), and not from sequences obtained from the maximization approaches. In other words, data from the maximization approaches will no longer be considered in the subsequent research questions.

5.2. *Research question 2*

The objective functions presented here result in an improvement over the previously published objective functions of McMullen *et al.* (2002). The approaches from the previously published work result in a mean ratio of 1.0279, while the work presented here results in an overall mean ratio of 1.0263. This difference is significant at the $\alpha = 0.05$ level ($F = 10.81, p = 0.001$).

5.3. *Research question 3*

Regarding the third research question, analysis shows that the objective function concerned with minimization of deterministic replacements clearly outperforms the other objective functions presented with this research. The minimal deterministic replacements objective function results in a mean ratio of 1.0222, while the other objective functions result in a mean ratio of 1.0277. This difference is significant at the $\alpha = 0.05$ level ($F = 62.97, p < 0.001$).

5.4. *Research question 4*

The third research question addressed above partially addresses the fourth research question—the minimal deterministic replacement approach is a relatively strong performer. Tables 9 and 10 detail this claim. For each table, the minimum mean number of required replacements is underlined.

Prob. set	Min. det. replace	Min. total gap	Max total gap	Min. std. gap	Max std. gap	Min gradient	Max gradient
1	1029.3	1030.0	1028.8	1028.7	1030.0	1029.2	1029.5
2	1028.6	1029.3	1029.5	1030.1	1028.4	1029.3	1030.1
3	2224.6	2223.3	2228.8	2224.2	2224.3	2228.0	2227.6
4	7276.6	7302.7	7364.2	7289.7	7297.0	7280.2	7292.6
5	8734.4	8739.6	8756.8	8750.0	8731.9	8740.9	8742.3
6	16962.9	17073.2	17196.1	17068.7	17303.7	17135.0	17068.3
7	8153.2	8152.0	8144.3	8152.3	8145.4	8149.6	8152.0
8	16118.6	16297.5	16353.6	16465.0	16735.6	16686.9	16535.6
9	23282.0	23631.8	23515.9	23561.5	23532.9	22800.2	23540.2
10	18573.9	18720.2	18723.1	18717.7	18786.9	18755.5	18731.5
11	38851.2	39953.3	39171.2	40547.1	39640.9	39512.0	39864.2

Table 9. Mean replacement by objective function for sequences obtained via simulated annealing.

Prob. set	Min. det. replace	Min. total gap	Max total gap	Min. std. gap	Max std. gap	Min gradient	Max gradient
1	1029.5	1029.6	1029.1	1030.1	1028.5	1029.3	1029.4
2	1029.4	1029.6	1029.3	1030.1	1028.8	1029.6	1029.5
3	2224.7	2224.6	2223.8	2224.1	2229.6	2228.6	2226.7
4	7280.5	7302.3	7288.7	7290.4	7296.3	7279.5	7294.0
5	8704.2	8741.5	8725.6	8749.6	8736.3	8741.2	8741.6
6	16964.4	17184.3	17340.6	17066.0	17303.4	17134.6	17065.5

Table 10. Mean replacement by objective function for sequences obtained via complete enumeration.

In several cases, the minimum deterministic replacements objective provides the most desirable result—especially for the larger problems. When the minimum deterministic replacements approach does not provide the most preferred result, it is typically close to 'the top of the list'. There are slight deviations from this, but no other pattern can be adequately explained.

5.5. *Research question 5*

From inspection and further analysis of tables 9 and 10, one can claim that when the objective function can be optimized via complete enumeration (as opposed to simulated annealing), preferred results are obtained. The mean ratio for solutions obtained via complete enumeration is 1.0231, while the mean ratio for solutions obtained via simulated annealing is 1.0280. This difference is significant at the $\alpha = 0.05$ level ($F = 61.03, p < 0.001$). While these significant differences are perhaps less than striking when inspecting tables 9 and 10, one can notice that the enumeration approach results in ratios that are consistently strong performers—while this approach may not consistently be at the 'top of the list', it never performs poorly either.

This particular finding essentially suggests that the objective functions are well motivated. When all solutions can be enumerated, the 'best' solution can be obtained in terms of its objective function value, and these values are relatively superior to those obtained when complete enumeration is not performed and/or not practical.

5.6. *Research question 6*

ANOVA shows that the coefficient of variation (*cv*) has a strong effect on replacement performance ($F = 1259.26$, $p < 0.001$). Table 11 shows that smaller values of cv result in relatively desirable performance.

Table 11 generally shows that higher levels of user-specified variation result in deteriorating replacement performance. Conversely, lower levels of variation result in superior performance—i.e. fewer replacements. It is also worth noting that the standard deviation for each of the three *cv* values also increases with simultaneous increases in the *cv* value.

Again, it is emphasized here that data obtained from the maximization approaches are not considered for Research questions 2–6, due to the relative inferiority of the objective function maximization approaches. Despite this, mean replacement data are shown in tables 9 and 10 for the maximization approaches only for purposes of full disclosure.

cv Value	Ratio Mean	Ratio Standard Deviation
0.05	1.0140	0.0161
0.15	1.0216	0.0180
0.25	1.0433	0.0187

Table 11. Breakdown of Performance by Coefficient of Variation Value (*cv*).

6. Concluding comments

Strategies, by way of objective functions, have been presented to find ways to sequence production such that tool wear is evenly distributed with the ultimate intent of saving money via fewer tooling replacements. In general, the results from this research effort are an improvement on a previous effort. In particular, the strategy involving minimal deterministic replacements has been found to be a desirable sequencing strategy with respect to the overall objective of minimizing tooling replacements. The deterministic approach of Objective function 1 is an effective way to find sequences with fewer required replacements despite the stochastic nature of the tool wear simulation. Unfortunately, the drawback of this particular strategy is that when the heuristic is searching for the best sequence, it is replicating the sequence ω times—this becomes expensive in terms of CPU time. The other presented objective function strategies, albeit relatively inferior to Objective function 1 in terms of required replacements (via the tool wear simulation), are much cheaper in terms of CPU time because the sequence is processed only once during the search process. It is also worth noting that the other objective function strategies presented here show that the minimization approach was found to be statistically superior to the maximization approach, but these differences pale in comparison to the relative superiority of the deterministic approach over the other approaches.

The author is well aware of the fact that simulated annealing is just one of many search approaches that could have been used for this research. Simulated annealing was used here because of its relative simplicity, as well as its generally well-documented strong performance. Other search heuristics could have been used here as well: genetic algorithms (Goldberg 1989, Michaelewicz 1990), tabu search (Glover 1990), artificial neural networks (Jain and Mao 1996) and ant-colony optimization approaches (Bonabeau *et al.*, 1999), just to name a few. It was not the intent of the author to compare various search heuristics here. The primary intent was to introduce the objective functions used to find sequences resulting in minimal replacements, and subsequently compare the performance results of these objective function strategies. Exploration of the various search heuristics (such as Tabu Search, etc) is an opportunity for future research. Another opportunity for future research is the development of additional objective functions dedicated to finding sequences minimizing tooling replacements.

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