

## TEACHING BRIEF

# Beyond Central Tendency: Helping Students Understand the Concept of Variation

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## INTRODUCTION

Getting business students to fully understand the concept of variation is often a major challenge for instructors of statistics/quantitative methods courses. As a result, it is all too common to see practicing managers placing too much emphasis on measures of central tendency (e.g., the mean and median) and not giving adequate attention to the variation inherent in the situation. Fundamentally, variation equates to the uncertainty associated with a particular decision. Without a full understanding and appreciation of the concept of variation, decision makers are ill-equipped to assess the risk associated with a given decision.

## EXAMPLE 1: TIGER WOODS VERSUS PHIL MICKELSON

In the second round of the 2005 British Open Championship, the world's two best golfers, Tiger Woods and Phil Mickelson, both shot scores of 67—five shots better than par. A score of 67 on The Old Course at St. Andrews is certainly a pleasing result to both of these established golfers. How these scores of 67 were obtained, however, is a different story for each golfer. The scorecard in Table 1 details how each golfer achieved their score of 67. The second row in the table lists the par for each hole. This is the score that a professional golfer is expected to make on each hole. One stroke better than par is called a “birdie,” and one stroke worse than par is called a “bogey.” In the scorecard, a score of birdie for the golfers is lightly shaded, while a score of bogey is more heavily shaded. For example, the 11th hole is one where Tiger earned a par, and Phil earned a birdie. Because each golfer shot a total score of 67, their average number of shots per hole is  $67/18 = 3.72$  ( $\bar{X} = 3.72$ ).

By comparing the mean shots per hole ( $\bar{X} = 3.72$ ) to each golfer's score on a particular hole ( $x_i$ ), we can compute their respective standard deviations via the following:

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**Table 1:** Scorecard for each golfer.

Hole	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Par	4	4	4	4	5	4	4	3	4	4	3	4	4	5	4	4	4	4
Tiger	4	4	3	4	4	4	4	3	3	3	3	4	4	4	4	4	4	4
Phil	5	3	4	4	4	4	5	2	3	4	2	3	4	4	4	4	5	3

**Table 2:** Squared differences with the mean for each golfer.

Hole	1	2	3	4	5	6	7	8	9
Tiger	.08	.08	.52	.08	.08	.08	.08	.52	.52
Phil	1.63	.52	.08	.08	.08	.08	1.63	2.97	.52
Hole	10	11	12	13	14	15	16	17	18
Tiger	.52	.52	.08	.08	.08	.08	.08	.08	.08
Phil	.08	2.97	.52	.08	.08	.08	.08	1.63	.52

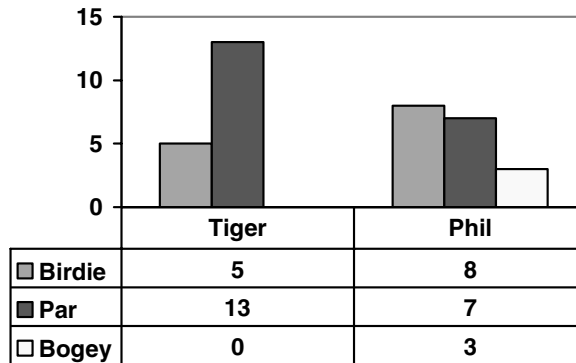
$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{X})^2}{n - 1}}$$

Intermediate detail of this calculation is shown in Table 2. That is, the squared difference term is shown for both golfers. For example, on the 11th hole, the squared difference in Tiger’s score with the mean is .52, while the squared difference between Phil’s score and the mean is 2.97.

This is where the comparison becomes interesting, and this is where students start to grasp understanding of the standard deviation calculation. From inspection of Table 2, notice that the squared differences for Tiger’s scores only take on two values: .08 and .52, while the squared difference for Phil’s scores take on four values, and some of them are quite large compared to Tiger’s. Phil shows more variation with respect to par than does Tiger. The 11th hole is a good example of this, but several other holes support this same finding. When the standard deviation for each golfer is computed, Tiger ends up with  $s = .4609$ , while Phil ends up with a standard deviation of  $s = .8948$ . Note that Phil’s standard deviation is roughly twice that of Tiger’s. For this in-class exercise, graphical representation is used to further reinforce the concept of variation via the histogram in Figure 1.

After presenting the histogram to the students, several interesting observations can be made. First of all, Tiger only had two different outcomes with respect to par: birdie and par. Phil had three different outcomes with respect to par: birdie, par, and bogey. In fact, Phil had more birdies than pars—a rather unusual result. Unfortunately for Phil, some of his birdie scores were offset by scores of bogey. In fact, Phil scored three bogeys. The fact that Phil had more birdies and more bogeys than did Tiger explains why Phil had a standard deviation that was roughly twice as high as Tiger. In short, Tiger is a model of consistency, while Phil is not, despite the

**Figure 1:** Histogram of performance for Tiger and Phil.



**Table 3:** Career statistics on Tiger Woods and Phil Mickelson (through 9/28/07).

Attribute	Tiger	Phil
Year turned pro	1996	1992
PGA victories	61	31
Major victories	13	3

fact that both of them shot a desirable score of 67. In concluding this example, it is instructive to have the students ponder the implications of being a more consistent golfer. In Tiger’s case avoiding bogeys meant there was less pressure on him to get the more difficult birdies. Certainly, Tiger was more consistent than Phil on this particular day, despite their aggregate performances being the same. Table 3 provides some interesting career statistics on Tiger and Phil. While risking extrapolation from a single round of golf, one could argue that consistency is conducive to long-term success, and Table 3 shows that Tiger’s long-term success is clearly superior to that of Phil.

**EXAMPLE 2: MAKING AN INVESTMENT DECISION**

Table 4 contains performance return data for two mutual funds: Gabelli Small Cap Growth AAA (GABSX) and Fidelity New Millennium (FMILX). Both funds invest in small and mid-cap stocks, focus on value and growth, and have over 95% of their assets invested in stocks. With this data set in hand, a relevant question to ask students is which fund they would choose to invest in and why.

If students behave like their real-world manager counterparts, they will tend to favor the FMILX fund, arguing that it averages a higher annual return. However, upon closer examination of the data, it can be observed that GABSX only lost money in 1 year out of 10 while FMILX lost money in 3 years. Furthermore, not only did FMILX lose money in three times as many years, its largest loss in any 1 year was 19.8% versus GABSX’s largest loss of only 5.3%. If an assumption

**Table 4:** Returns for two mutual funds.

Year	'05	'04	'03	'02	'01	'00	'99	'98	'97	'96	Avg.
	Return (%)										
GABSX	5.9	21.6	37.5	-5.3	4.6	11.3	14.2	.0	36.4	11.8	13.0
FMILX	10.1	4.2	37.3	-19.8	-18.1	-6.0	108.7	27.7	24.6	23.1	14.8

Source: *The Top Mutual Funds*, The American Association of Individual Investors 25th Edition, 2006.

of equal weighting/year is made, GABSX has a standard deviation of 14.32%, while FMILX has a standard deviation of 36.98%. The difference between these two investment options can be further illustrated using the Empirical Rule where it can be shown that in approximately 95% of the years GABSX's returns would be between -15.64% and 41.64% while FMILX would return between -59.16% and 88.76% in 95% of the years. The point of this example is that, by only focusing on the average returns, investors would likely have a preference for the FMILX fund. However, the more appropriate question in this situation is whether the extra 2% in annual returns is adequate compensation for the increased risk (risk is measured here by standard deviation) of losses associated with the FMILX fund. Of course, that decision must be made by the individual investor, who would be wise to consider the variation inherent in each investment option.

## CONCLUSION

We have found these examples to be useful in helping students better understand the concept of variation and the importance of considering it in decision making. In recent years, the use of these examples has always been well received by students, and students have repeatedly supplied feedback regarding the value of the exercises. This positive feedback has been in two forms: (1) official feedback via the course evaluation system and (2) informal feedback via e-mail and conversations with students, where the students consistently articulate the value of these exercises.

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